New Tools for Scalable Weighted Sampling: Frequency Capping, Multi-Objective, and more

Edith Cohen

[1]Google, CA USA

[2]School of Computer Science
Tel Aviv University, Israel

October 18, 2015

Data Model

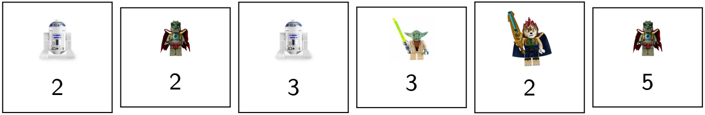
Data *elements* (x, w) have a *key* x and a numeric *value* $w > 0$

- Elements are streamed or distributed, no particular order/partition
- "Unaggregated:" Multiple elements can have the same key
- "Aggregated:" Elements have unique keys

Data Model

Data *elements* (x, w) have a *key* x and a numeric *value* $w > 0$

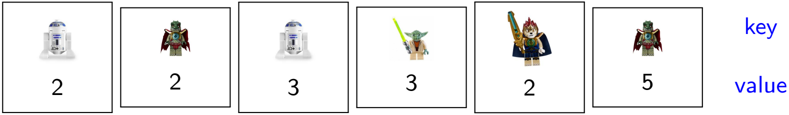
- Elements are streamed or distributed, no particular order/partition
- "Unaggregated:" Multiple elements can have the same key
- "Aggregated:" Elements have unique keys



Data Model

Data *elements* (x, w) have a *key* x and a numeric *value* $w > 0$

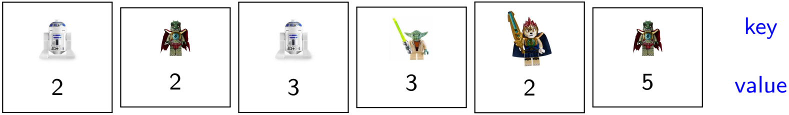
- Elements are streamed or distributed, no particular order/partition
- "Unaggregated:" Multiple elements can have the same key
- "Aggregated:" Elements have unique keys



Data Model

Data *elements* (x, w) have a *key* x and a numeric *value* $w > 0$

- Elements are streamed or distributed, no particular order/partition
- "Unaggregated:" Multiple elements can have the same key
- "Aggregated:" Elements have unique keys

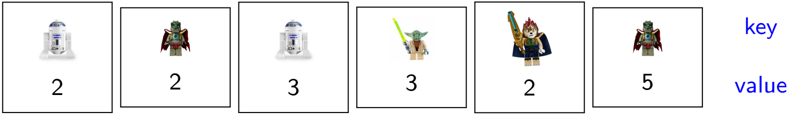


The *aggregated view* of unaggregated data: The set of key value pairs (x, w_x) for active keys x. w_x is the sum of values of elements with key x.

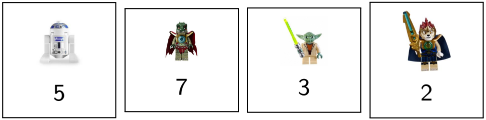
Data Model

Data *elements* (x, w) have a *key* x and a numeric *value* $w > 0$

- Elements are streamed or distributed, no particular order/partition
- "Unaggregated:" Multiple elements can have the same key
- "Aggregated:" Elements have unique keys



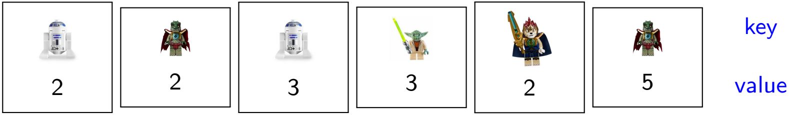
The *aggregated view* of unaggregated data: The set of key value pairs (x, w_x) for active keys x. w_x is the sum of values of elements with key x.



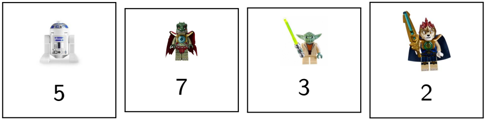
Data Model

Data *elements* (x, w) have a *key* x and a numeric *value* $w > 0$

- Elements are streamed or distributed, no particular order/partition
- "Unaggregated:" Multiple elements can have the same key
- "Aggregated:" Elements have unique keys



The *aggregated view* of unaggregated data: The set of key value pairs (x, w_x) for active keys x. w_x is the sum of values of elements with key x.



Queries are typically specified over the aggregated view

Scalable Computation

One (or few) passes over the data

- Streaming (single sequential pass): Necessary for live dashboards and when data is discarded. Historically model captured sequential-access storage devices (tape, disks), Unix pipes. Streaming model: [Knu68], [MG82], [FM85],. . ., formalized in [AMS99]
- Distributed/Parallel aggregation: Process parts of the data and combine small summaries (look at each part once or few times)

Scalable Computation

One (or few) passes over the data

- Streaming (single sequential pass): Necessary for live dashboards and when data is discarded. Historically model captured sequential-access storage devices (tape, disks), Unix pipes. Streaming model: [Knu68], [MG82], [FM85],..., formalized in [AMS99]
- Distributed/Parallel aggregation: Process parts of the data and combine small summaries (look at each part once or few times)

Small state

- When streaming, the state is what we keep in memory
- In distributed aggregation, it is the summary size that is shared

We want state $\ll$ number of (distinct) keys

Scalable Computation

One (or few) passes over the data

- Streaming (single sequential pass): Necessary for live dashboards and when data is discarded. Historically model captured sequential-access storage devices (tape, disks), Unix pipes. Streaming model: [Knu68], [MG82], [FM85],..., formalized in [AMS99]
- Distributed/Parallel aggregation: Process parts of the data and combine small summaries (look at each part once or few times)

Small state

- When streaming, the state is what we keep in memory
- In distributed aggregation, it is the summary size that is shared

We want state $\ll$ number of (distinct) keys

Challenge with unaggregated data: Computing the aggregated view $\{(x, w_x)\}$ requires state $\propto$ number of active keys, which can be very large.

Frequency statistics

$$Q(f, H) = \sum_{x \in H} f(w_x)$$

- Function $f(w) \geq 0$ for $w \geq 0$ so that $f(0) = 0$, usually monotone non-decreasing
- Selected *segment* $H \subset \mathcal{X}$ (domain, subpopulation) from all keys

Example $f()$:

Frequency statistics

$$Q(f, H) = \sum_{x \in H} f(w_x)$$

- Function $f(w) \geq 0$ for $w \geq 0$ so that $f(0) = 0$, usually monotone non-decreasing
- Selected *segment* $H \subset \mathcal{X}$ (domain, subpopulation) from all keys

Example $f()$:

- Distinct $f(w) = 1$ (# active keys in segment)

Frequency statistics

$$Q(f, H) = \sum_{x \in H} f(w_x)$$

- Function $f(w) \geq 0$ for $w \geq 0$ so that $f(0) = 0$, usually monotone non-decreasing
- Selected *segment* $H \subset \mathcal{X}$ (domain, subpopulation) from all keys

Example $f()$:

- Distinct $f(w) = 1$ (# active keys in segment)
- Sum $f(w) = w$ (sum of weights of keys in segment)

Frequency statistics

$$Q(f, H) = \sum_{x \in H} f(w_x)$$

- Function $f(w) \geq 0$ for $w \geq 0$ so that $f(0) = 0$, usually monotone non-decreasing
- Selected *segment* $H \subset \mathcal{X}$ (domain, subpopulation) from all keys

Example $f()$:

- Distinct $f(w) = 1$ (# active keys in segment)
- Sum $f(w) = w$ (sum of weights of keys in segment)
- Moments $f(w) = w^p$ $(p \geq 0)$ (distinct $p = 0$, sum $p = 1$)

Frequency statistics

$$Q(f,H) = \sum_{x\in H} f(w_x)$$

- Function $f(w) \geq 0$ for $w \geq 0$ so that $f(0) = 0$, usually monotone non-decreasing
- Selected *segment* $H \subset \mathcal{X}$ (domain, subpopulation) from all keys

Example $f()$:

- Distinct $f(w) = 1$ (# active keys in segment)
- Sum $f(w) = w$ (sum of weights of keys in segment)
- Moments $f(w) = w^p$ $(p \geq 0)$ (distinct $p = 0$, sum $p = 1$)
- Capping $f(w) = \mathrm{cap}_T = \min\{T, w\}$ (distinct $T = 1$, sum $T = +\infty$)

Frequency statistics

$$Q(f, H) = \sum_{x \in H} f(w_x)$$

- Function $f(w) \geq 0$ for $w \geq 0$ so that $f(0) = 0$, usually monotone non-decreasing
- Selected *segment* $H \subset \mathcal{X}$ (domain, subpopulation) from all keys

Example $f()$:

- Distinct $f(w) = 1$ (# active keys in segment)
- Sum $f(w) = w$ (sum of weights of keys in segment)
- Moments $f(w) = w^p$ $(p \geq 0)$ (distinct $p = 0$, sum $p = 1$)
- Capping $f(w) = \text{cap}_T = \min\{T, w\}$ (distinct $T = 1$, sum $T = +\infty$)
- Threshold $f(w) = \text{thresh}_T = I_{w \geq T}$ $(T > 0)$

Frequency statistics

$$Q(f, H) = \sum_{x \in H} f(w_x)$$

- Function $f(w) \geq 0$ for $w \geq 0$ so that $f(0) = 0$, usually monotone non-decreasing
- Selected *segment* $H \subset \mathcal{X}$ (domain, subpopulation) from all keys

Example $f()$:

- Distinct $f(w) = 1$ (# active keys in segment)
- Sum $f(w) = w$ (sum of weights of keys in segment)
- Moments $f(w) = w^p$ $(p \geq 0)$ (distinct $p = 0$, sum $p = 1$)
- Capping $f(w) = \text{cap}_T = \min\{T, w\}$ (distinct $T = 1$, sum $T = +\infty$)
- Threshold $f(w) = \text{thresh}_T = I_{w \geq T}$ $(T > 0)$

Moments w^p with $p \in [0, 1]$ and cap statistics cap_T with $T \in (0, +\infty)$ parametrize the range between distinct and sum.

Use case for frequency capping: Online advertising

The first few impressions of the same ad per user are more effective than later ones (diminishing return). Advertisers therefore specify

- A segment of users (based on geography, demographics, other)
- Cap T on the number of impressions per user per time period.

Use case for frequency capping: Online advertising

The first few impressions of the same ad per user are more effective than later ones (diminishing return). Advertisers therefore specify

- A segment of users (based on geography, demographics, other)
- Cap T on the number of impressions per user per time period.

1,567,856

22

89

121

2

Use case for frequency capping: Online advertising

The first few impressions of the same ad per user are more effective than later ones (diminishing return). Advertisers therefore specify

- A segment of users (based on geography, demographics, other)
- Cap T on the number of impressions per user per time period.


1,567,856


22


89


121


2

Q: targeted segment: galactic-scale travelers cap: 5

Use case for frequency capping: Online advertising

The first few impressions of the same ad per user are more effective than later ones (diminishing return). Advertisers therefore specify

- A segment of users (based on geography, demographics, other)
- Cap T on the number of impressions per user per time period.

1,567,856

22

89

121

2

Q: targeted segment: galactic-scale travelers cap: 5
Answer (number of qualifying impressions): 15

Use case for frequency capping: Online advertising

The first few impressions of the same ad per user are more effective than later ones (diminishing return). Advertisers therefore specify

- A segment of users (based on geography, demographics, other)
- Cap T on the number of impressions per user per time period.

 1,567,856	 22	 89	 121	 2

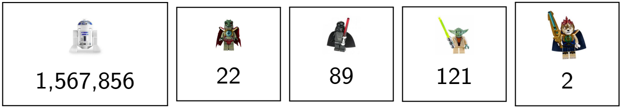
Q: targeted segment: galactic-scale travelers cap: 5
Answer (number of qualifying impressions): 15

Q: targeted segment: non-human intelligent life cap: 3

Use case for frequency capping: Online advertising

The first few impressions of the same ad per user are more effective than later ones (diminishing return). Advertisers therefore specify

- A segment of users (based on geography, demographics, other)
- Cap T on the number of impressions per user per time period.



Q: targeted segment: galactic-scale travelers   cap: 5
Answer (number of qualifying impressions): 15

Q: targeted segment: non-human intelligent life   cap: 3
Answer (number of qualifying impressions): 8

… Frequency Capping in Online advertising

Advertisers specify:

- A segment H of users (based on geography, demographics, other)
- A cap T on the number of impressions per user per time period.

... Frequency Capping in Online advertising

Advertisers specify:

- A segment H of users (based on geography, demographics, other)
- A cap T on the number of impressions per user per time period.

Campaign planning is interactive. Staging tools use past data to predict the number $Q(\text{cap}_T, H)$ of qualifying impressions.

- Data is "unaggregated:" Impressions for same user come from diverse sources (devices, apps, times)

... Frequency Capping in Online advertising

Advertisers specify:

- A segment H of users (based on geography, demographics, other)
- A cap T on the number of impressions per user per time period.

Campaign planning is interactive. Staging tools use past data to predict the number $Q(\text{cap}_T, H)$ of qualifying impressions.

- Data is "unaggregated:" Impressions for same user come from diverse sources (devices, apps, times)

$\Longrightarrow$ Need quick estimates $\hat{Q}(\text{cap}_T, H)$ from a summary that is computed efficiently over the unaggregated data set.

Frequency statistics challenges

Multi-objective sample (un)aggregated data: For a set of functions F, compute a summary/sample from which we can *estimate* $Q(f, H)$ for various $f \in F$, $H \subseteq \mathcal{X}$.

Frequency statistics challenges

Multi-objective sample (un)aggregated data: For a set of functions F, compute a summary/sample from which we can *estimate* $Q(f, H)$ for various $f \in F$, $H \subseteq \mathcal{X}$.

Weighted sample unaggregated data: For a given f, compute a summary/sample from which we can *estimate* $Q(f, H)$ for various H

- Basic: Estimate $Q(f, H)$ for a given f, $H \subseteq \mathcal{X}$

Frequency statistics challenges

Multi-objective sample (un)aggregated data: For a set of functions F, compute a summary/sample from which we can *estimate* $Q(f, H)$ for various $f \in F$, $H \subseteq \mathcal{X}$.

Weighted sample unaggregated data: For a given f, compute a summary/sample from which we can *estimate* $Q(f, H)$ for various H

- Basic: Estimate $Q(f, H)$ for a given f, $H \subseteq \mathcal{X}$

Goals: • Optimize tradeoffs of sample quality (statistical guarantees) and size. • Scalable computation.

Frequency statistics challenges

Multi-objective sample (un)aggregated data: For a set of functions F, compute a summary/sample from which we can *estimate* $Q(f, H)$ for various $f \in F$, $H \subseteq \mathcal{X}$.

Weighted sample unaggregated data: For a given f, compute a summary/sample from which we can *estimate* $Q(f, H)$ for various H

- Basic: Estimate $Q(f, H)$ for a given f, $H \subseteq \mathcal{X}$

Goals: • Optimize tradeoffs of sample quality (statistical guarantees) and size. • Scalable computation.

Plan for this talk:

Frequency statistics challenges

Multi-objective sample (un)aggregated data: For a set of functions F, compute a summary/sample from which we can *estimate* $Q(f, H)$ for various $f \in F$, $H \subseteq \mathcal{X}$.

Weighted sample unaggregated data: For a given f, compute a summary/sample from which we can *estimate* $Q(f, H)$ for various H

- Basic: Estimate $Q(f, H)$ for a given f, $H \subseteq \mathcal{X}$

Goals: • Optimize tradeoffs of sample quality (statistical guarantees) and size. • Scalable computation.

Plan for this talk:

- *Aggregated data sets*: Review the "gold standard" Sample size/ estimation quality tradeoffs. • Multi-objective sampling scheme for all monotone (non-decreasing) f. [Coh15b]

Frequency statistics challenges

Multi-objective sample (un)aggregated data: For a set of functions F, compute a summary/sample from which we can *estimate* $Q(f, H)$ for various $f \in F$, $H \subseteq \mathcal{X}$.

Weighted sample unaggregated data: For a given f, compute a summary/sample from which we can *estimate* $Q(f, H)$ for various H

- Basic: Estimate $Q(f, H)$ for a given f, $H \subseteq \mathcal{X}$

Goals: • Optimize tradeoffs of sample quality (statistical guarantees) and size. • Scalable computation.

Plan for this talk:

- *Aggregated data sets*: Review the "gold standard" Sample size/ estimation quality tradeoffs. • Multi-objective sampling scheme for all monotone (non-decreasing) f. [Coh15b]
- Unaggregated data sets: How to sample effectively *without* aggregation for capping statistics (and more) [Coh15c]

Aggregated data: Weighted sampling schemes

- Data provided as key value pairs (x, w_x).
- Compute a sample S_f of size k from which we can estimate $Q(f, H)$.

Aggregated data: Weighted sampling schemes

- Data provided as key value pairs (x, w_x).
- Compute a sample S_f of size k from which we can estimate $Q(f, H)$.

To get good size/quality tradeoffs, need (roughly) $\Pr[x \in S] \propto f(w_x)$:

Aggregated data: Weighted sampling schemes

- Data provided as key value pairs (x, w_x).
- Compute a sample S_f of size k from which we can estimate $Q(f, H)$.

To get good size/quality tradeoffs, need (roughly) $\Pr[x \in S] \propto f(w_x)$:

- Poisson Probability Proportional to Size (PPS): Sample keys independently with $p_x = \min\{1, \frac{kf(w_x)}{\sum_x f(w_x)}\}$
- VarOpt [Cha82, CDL+11]: Dependent PPS for sample size exactly k

Aggregated data: Weighted sampling schemes

- Data provided as key value pairs (x, w_x).
- Compute a sample S_f of size k from which we can estimate $Q(f, H)$.

To get good size/quality tradeoffs, need (roughly) $\Pr[x \in S] \propto f(w_x)$:

- Poisson Probability Proportional to Size (PPS): Sample keys independently with $p_x = \min\{1, \frac{kf(w_x)}{\sum_x f(w_x)}\}$
- VarOpt [Cha82, CDL+11]: Dependent PPS for sample size exactly k

Bottom-k/order/weighted reservoir sampling schemes [Ros97, CK07]

foreach *key* x **do** // $Z[w]$: distribution parameterized by w
  seed$(x) \sim Z[f(w_x)]$

$S \leftarrow k$ keys with smallest seed(x); $\tau \leftarrow (k+1)$th smallest seed(x)

Aggregated data: Weighted sampling schemes

- Data provided as key value pairs (x, w_x).
- Compute a sample S_f of size k from which we can estimate $Q(f, H)$.

To get good size/quality tradeoffs, need (roughly) $\Pr[x \in S] \propto f(w_x)$:

- Poisson Probability Proportional to Size (PPS): Sample keys independently with $p_x = \min\{1, \frac{kf(w_x)}{\sum_x f(w_x)}\}$
- VarOpt [Cha82, CDL+11]: Dependent PPS for sample size exactly k

Bottom-k/order/weighted reservoir sampling schemes [Ros97, CK07]

foreach *key* x **do** // $Z[w]$: distribution parameterized by w
  $\mathtt{seed}(x) \sim Z[f(w_x)]$
$S \leftarrow k$ keys with smallest $\mathtt{seed}(x)$; $\tau \leftarrow (k+1)$th smallest $\mathtt{seed}(x)$

- Sequential Poisson (priority) [Ohl98, DTL07]:
  $\mathtt{seed}(x) \sim U[0, 1/f(w_x)]$
- PPS without replacement (ppswor) [Ros72, Coh97, CK07]:
  $\mathtt{seed}(x) \sim \mathrm{Exp}[f(w_x)]$

Aggregated: More on pps without replacement (ppswor)

Two equivalent formulations [Ros72]

foreach *key* x **do**
  `seed`$(x) \sim \text{Exp}[f(w_x)]$
$S \leftarrow k$ keys with smallest `seed`(x)

$S \leftarrow \emptyset$
repeat
  Sample $x \notin S$ using

$$p_x = \frac{f(w_x)}{\sum_{y \notin S} f(w_y)}$$

  $S \leftarrow S \cup \{x\}$
until $|S| = k$;

Aggregated: More on pps without replacement (ppswor)

Two equivalent formulations [Ros72]

foreach *key* x **do**
  $\texttt{seed}(x) \sim \mathrm{Exp}[f(w_x)]$
$S \leftarrow k$ keys with smallest $\texttt{seed}(x)$

$S \leftarrow \emptyset$
repeat
  Sample $x \notin S$ using

$$p_x = \frac{f(w_x)}{\sum_{y \notin S} f(w_y)}$$

  $S \leftarrow S \cup \{x\}$
until $|S| = k$;

We focus on ppswor:

- Similar (near optimal) sample size/quality tradeoffs to other weighted sampling schemes
- Our proposed schemes for unaggregated data build on ppswor

Aggregated data: Estimators for weighted samples

Inverse probability estimator of $Q(g, H)$ from the sample S [HT52]

$p_x = \Pr[x \in S]$: probability that key x is sampled
For each key x, estimate $g(w_x)$ by 0 if $x \notin S$ and by $g(w_x)/p_x$ if $x \in S$.

Aggregated data: Estimators for weighted samples

Inverse probability estimator of $Q(g, H)$ from the sample S [HT52]

$p_x = \Pr[x \in S]$: probability that key x is sampled
For each key x, estimate $g(w_x)$ by 0 if $x \notin S$ and by $g(w_x)/p_x$ if $x \in S$.

$$\hat{Q}(g, H) = \sum_{x \in H} \hat{g}(w_x) = \sum_{x \in H \cap S} \frac{g(w_x)}{p_x} .$$

Aggregated data: Estimators for weighted samples

Inverse probability estimator of $Q(g, H)$ from the sample S [HT52]

$p_x = \Pr[x \in S]$: probability that key x is sampled
For each key x, estimate $g(w_x)$ by 0 if $x \notin S$ and by $g(w_x)/p_x$ if $x \in S$.

$$\hat{Q}(g, H) = \sum_{x \in H} \hat{g}(w_x) = \sum_{x \in H \cap S} \frac{g(w_x)}{p_x} .$$

Applies when we can compute p_x for $x \in S$

Aggregated data: Estimators for weighted samples

Inverse probability estimator of $Q(g, H)$ from the sample S [HT52]

$p_x = \Pr[x \in S]$: probability that key x is sampled
For each key x, estimate $g(w_x)$ by 0 if $x \notin S$ and by $g(w_x)/p_x$ if $x \in S$.

$$\hat{Q}(g, H) = \sum_{x \in H} \hat{g}(w_x) = \sum_{x \in H \cap S} \frac{g(w_x)}{p_x} .$$

Applies when we can compute p_x for $x \in S$

- nonnegative (since g is)
- unbiased (if $g(w_x) > 0 \implies f(w_x) > 0$)

Aggregated data: Estimators for weighted samples

Inverse probability estimator of $Q(g, H)$ from the sample S [HT52]

$p_x = \Pr[x \in S]$: probability that key x is sampled
For each key x, estimate $g(w_x)$ by 0 if $x \notin S$ and by $g(w_x)/p_x$ if $x \in S$.

$$\hat{Q}(g, H) = \sum_{x \in H} \hat{g}(w_x) = \sum_{x \in H \cap S} \frac{g(w_x)}{p_x} .$$

Applies when we can compute p_x for $x \in S$

- nonnegative (since g is)
- unbiased (if $g(w_x) > 0 \implies f(w_x) > 0$)

Poisson PPS samples: $p_x = \min\{1, \frac{kf(w_x)}{\sum_x f(w_x)}\}$
We have w_x for sampled keys $x \in S$, and the total $\sum_x f(w_x)$
$\implies$ can compute p_x and apply estimator.

Aggregated data: Estimators for weighted samples

Inverse probability estimator of $Q(g, H)$ from the sample S [HT52]

$p_x = \Pr[x \in S]$: probability that key x is sampled
For each key x, estimate $g(w_x)$ by 0 if $x \notin S$ and by $g(w_x)/p_x$ if $x \in S$.

$$\hat{Q}(g, H) = \sum_{x \in H} \hat{g}(w_x) = \sum_{x \in H \cap S} \frac{g(w_x)}{p_x} .$$

Applies when we can compute p_x for $x \in S$

- nonnegative (since g is)
- unbiased (if $g(w_x) > 0 \implies f(w_x) > 0$)

Bottom-k samples: p_x is not available so instead we use

$$p_{x|\tau} \equiv \Pr[\mathrm{seed}(x) < \tau] = \Pr[Z[f(w_x)] < \tau]$$

Aggregated data: Estimators for weighted samples

Inverse probability estimator of $Q(g, H)$ from the sample S [HT52]

$p_x = \Pr[x \in S]$: probability that key x is sampled
For each key x, estimate $g(w_x)$ by 0 if $x \notin S$ and by $g(w_x)/p_x$ if $x \in S$.

$$\hat{Q}(g, H) = \sum_{x \in H} \hat{g}(w_x) = \sum_{x \in H \cap S} \frac{g(w_x)}{p_x} .$$

Applies when we can compute p_x for $x \in S$

- nonnegative (since g is)
- unbiased (if $g(w_x) > 0 \implies f(w_x) > 0$)

Bottom-k samples: p_x is not available so instead we use

$$p_{x|\tau} \equiv \Pr[\texttt{seed}(x) < \tau] = \Pr[Z[f(w_x)] < \tau]$$

The inclusion probability of x *conditioned* on randomization of all other keys: τ is the kth smallest $\texttt{seed}(y)$ for $y \neq x$; $x \in S \iff \texttt{seed}(x) < \tau$

- For ppswor $Z[y] \equiv \text{Exp}[y]$: $p_{x|\tau} = 1 - e^{-f(w_x)\tau}$
- For priority $Z[y] \equiv U[0, 1/y]$: $p_{x|\tau} = \min\{f(w_x)\tau, 1\}$

Aggregated data: Estimators for weighted samples

Inverse probability estimator of $Q(g, H)$ from the sample S [HT52]

$p_x = \Pr[x \in S]$: probability that key x is sampled
For each key x, estimate $g(w_x)$ by 0 if $x \notin S$ and by $g(w_x)/p_x$ if $x \in S$.

$$\hat{Q}(g, H) = \sum_{x \in H} \hat{g}(w_x) = \sum_{x \in H \cap S} \frac{g(w_x)}{p_x} .$$

Applies when we can compute p_x for $x \in S$

- nonnegative (since g is)
- unbiased (if $g(w_x) > 0 \implies f(w_x) > 0$)

Bottom-k samples: p_x is not available so instead we use

$$p_{x|\tau} \equiv \Pr[\text{seed}(x) < \tau] = \Pr[Z[f(w_x)] < \tau]$$

Aggregated data: Estimators for weighted samples

Inverse probability estimator of $Q(g, H)$ from the sample S [HT52]

$p_x = \Pr[x \in S]$: probability that key x is sampled
For each key x, estimate $g(w_x)$ by 0 if $x \notin S$ and by $g(w_x)/p_x$ if $x \in S$.

$$\hat{Q}(g, H) = \sum_{x \in H} \hat{g}(w_x) = \sum_{x \in H \cap S} \frac{g(w_x)}{p_x} .$$

Applies when we can compute p_x for $x \in S$

- nonnegative (since g is)
- unbiased (if $g(w_x) > 0 \implies f(w_x) > 0$)

Bottom-k samples: p_x is not available so instead we use

$$p_{x|\tau} \equiv \Pr[\text{seed}(x) < \tau] = \Pr[Z[f(w_x)] < \tau]$$

$$\hat{Q}(g, H) = \sum_{x \in H \cap S} \hat{g}(w_x \mid \tau) , \text{ where } \hat{g}(w_x \mid \tau) = \frac{g(w_x)}{p_{x|\tau}} .$$

How good is this estimate?

Aggregated: ppswor estimate quality when $g() = f()$

Let $q \equiv q(f, H)$ be the fraction of the statistics f due to segment H:

$$q = \frac{Q(f, H)}{Q(f, \mathcal{X})} = \frac{\sum_{x \in H} f(w_x)}{\sum_x f(w_x)} \ .$$

Aggregated: ppswor estimate quality when $g() = f()$

Let $q \equiv q(f, H)$ be the fraction of the statistics f due to segment H:

$$q = \frac{Q(f,H)}{Q(f,\mathcal{X})} = \frac{\sum_{x \in H} f(w_x)}{\sum_x f(w_x)} .$$

bound on the Coefficient of Variation (CV) (relative standard deviation)

$$\frac{\sqrt{var[\hat{Q}(f,H)]}}{Q(f,H)} \leq \frac{1}{\sqrt{q(k-1)}}$$

Aggregated: ppswor estimate quality when $g() = f()$

Let $q \equiv q(f, H)$ be the fraction of the statistics f due to segment H:

$$q = \frac{Q(f,H)}{Q(f,\mathcal{X})} = \frac{\sum_{x \in H} f(w_x)}{\sum_x f(w_x)} .$$

bound on the Coefficient of Variation (CV) (relative standard deviation)

$$\frac{\sqrt{var[\hat{Q}(f,H)]}}{Q(f,H)} \leq \frac{1}{\sqrt{q(k-1)}}$$

+concentration: sample size $k = c\epsilon^{-2}/q$ then prob. of rel. error $> \epsilon$ decreases exponentially in c.

Aggregated: Interpreting the CV bound for $g() = f()$

CV (relative standard deviation, NRMSE) bound

$$\frac{\sqrt{var[\hat{Q}(f,H)]}}{Q(f,H)} \leq \frac{1}{\sqrt{q(k-1)}}$$

Aggregated: Interpreting the CV bound for $g() = f()$

CV (relative standard deviation, NRMSE) bound

$$\frac{\sqrt{var[\hat{Q}(f,H)]}}{Q(f,H)} \leq \frac{1}{\sqrt{q(k-1)}}$$

$\Longrightarrow$ If we want CV $\leq \epsilon$ on segments H that have $q(f,H) \geq q$ fraction of the total f statistics, we need a sample of size $k = \epsilon^{-2}/q$

Aggregated: Interpreting the CV bound for $g() = f()$

CV (relative standard deviation, NRMSE) bound

$$\frac{\sqrt{var[\hat{Q}(f,H)]}}{Q(f,H)} \leq \frac{1}{\sqrt{q(k-1)}}$$

$\Longrightarrow$ If we want CV $\leq \epsilon$ on segments H that have $q(f,H) \geq q$ fraction of the total f statistics, we need a sample of size $k = \epsilon^{-2}/q$

!! This is the optimal size/quality tradeoff for sampling (on average over segments with proportion q)

1,567,856

22

89

121

2

For CV $\epsilon \leq 10\%$ and $q \geq 0.1\%$ $\Longrightarrow$ Sample size $k = 10^5$.

Aggregated: Interpreting the CV bound for $g() = f()$

CV (relative standard deviation, NRMSE) bound

$$\frac{\sqrt{var[\hat{Q}(f,H)]}}{Q(f,H)} \leq \frac{1}{\sqrt{q(k-1)}}$$

$\Longrightarrow$ If we want CV $\leq \epsilon$ on segments H that have $q(f,H) \geq q$ fraction of the total f statistics, we need a sample of size $k = \epsilon^{-2}/q$

!! This is the optimal size/quality tradeoff for sampling (on average over segments with proportion q)

 1,567,856	 22	 89	 121	 2

For CV $\epsilon \leq 10\%$ and $q \geq 0.1\%$ $\Longrightarrow$ Sample size $k = 10^5$.

. . . usually $k \ll$ total number of active keys.

Aggregated: ppswor estimate quality when $g() \neq f()$

What can we say about estimate quality when $g() \neq f()$?

Aggregated: ppswor estimate quality when $g() \neq f()$

What can we say about estimate quality when $g() \neq f()$?

Disparity between g, f:

$$\rho(g, f) = \max_{w>0} \frac{g(w)}{f(w)} \max_{w>0} \frac{f(w)}{g(w)} \ .$$

Aggregated: ppswor estimate quality when $g() \neq f()$

What can we say about estimate quality when $g() \neq f()$?

Disparity between g, f:

$$\rho(g,f) = \max_{w>0} \frac{g(w)}{f(w)} \max_{w>0} \frac{f(w)}{g(w)} \ .$$

- Disparity is always $\rho(g,f) \geq 1$.
- We have $\rho(g,f) = 1 \iff g = cf$ for some c.

Aggregated: ppswor estimate quality when $g() \neq f()$

What can we say about estimate quality when $g() \neq f()$?

Disparity between g, f:

$$\rho(g,f) = \max_{w>0} \frac{g(w)}{f(w)} \max_{w>0} \frac{f(w)}{g(w)} \ .$$

- Disparity is always $\rho(g,f) \geq 1$.
- We have $\rho(g,f) = 1 \iff g = cf$ for some c.

Lemma

CV of $\hat{Q}(g,H)$ is at most $\left(\frac{\rho}{q(k-1)}\right)^{0.5}$.

Aggregated: Proof of variance bound for sample size k

$$\begin{aligned}
var[\hat{g}(w_x \mid \tau)] &= \mathrm{E}[(\hat{g}(w_x \mid \tau))^2] - g(w_x)^2 \\
&= p_{x|\tau}\frac{g(w_x)^2}{p_{x|\tau}^2} + (1 - p_{x|\tau}) \cdot 0 - g(w_x)^2 \\
&= \left(\frac{1}{p_{x|\tau}} - 1\right) g(w_x)^2 \\
&< g(w_x)^2 \frac{e^{-\tau f(w_x)}}{1 - e^{-\tau f(w_x)}} \leq \frac{1}{\tau f(w_x)} g(w_x)^2 \leq \max_{w>0} \frac{f(w)}{g(w)} \frac{g(w_x)}{\tau} \ .
\end{aligned}$$

Aggregated: Proof of variance bound for sample size k

$$
\begin{aligned}
var[\hat{g}(w_x \mid \tau)] &= \mathrm{E}[(\hat{g}(w_x \mid \tau))^2] - g(w_x)^2 \\
&= p_{x|\tau}\frac{g(w_x)^2}{p_{x|\tau}^2} + (1-p_{x|\tau})\cdot 0 - g(w_x)^2 \\
&= \left(\frac{1}{p_{x|\tau}} - 1\right) g(w_x)^2 \\
&< g(w_x)^2 \frac{e^{-\tau f(w_x)}}{1-e^{-\tau f(w_x)}} \leq \frac{1}{\tau f(w_x)} g(w_x)^2 \leq \max_{w>0}\frac{f(w)}{g(w)}\frac{g(w_x)}{\tau} .
\end{aligned}
$$

We take expectation over distribution of τ, which is dominated by Erlang (sum of k independent $\mathrm{Exp}(\sum_y f(w_y))$)

$$
var[\hat{g}(w_x)] \leq \mathrm{E}_{\tau\sim \mathrm{Erlang}} var[\hat{g}(w_x) \mid \tau] \leq \max_{w>0}\frac{f(w)}{g(w)}\frac{g(w_x)}{(k-1)\sum_y f(w_y)} .
$$

Aggregated: Proof of variance bound for sample size k

$$\begin{aligned}
var[\hat{g}(w_x \mid \tau)] &= \mathsf{E}[(\hat{g}(w_x \mid \tau))^2] - g(w_x)^2 \\
&= p_{x|\tau}\frac{g(w_x)^2}{p_{x|\tau}^2} + (1-p_{x|\tau})\cdot 0 - g(w_x)^2 \\
&= \left(\frac{1}{p_{x|\tau}} - 1\right) g(w_x)^2 \\
&< g(w_x)^2 \frac{e^{-\tau f(w_x)}}{1-e^{-\tau f(w_x)}} \leq \frac{1}{\tau f(w_x)} g(w_x)^2 \leq \max_{w>0}\frac{f(w)}{g(w)}\frac{g(w_x)}{\tau} .
\end{aligned}$$

We take expectation over distribution of τ, which is dominated by Erlang (sum of k independent $\mathsf{Exp}(\sum_y f(w_y))$)

$$var[\hat{g}(w_x)] \leq \mathsf{E}_{\tau\sim\mathsf{Erlang}} var[\hat{g}(w_x) \mid \tau] \leq \max_{w>0}\frac{f(w)}{g(w)}\frac{g(w_x)}{(k-1)\sum_y f(w_y)} .$$

We use zero covariances to obtain
$var[\hat{Q}(g,H)] = \sum_{x\in H} var[\hat{g}(w_x)] \leq \max_{w>0}\frac{f(w)}{g(w)}\frac{1}{k-1}\frac{\sum_{x\in H} g(w_x)}{\sum_x f(w_x)} \leq \frac{\rho}{q(k-1)}$

Aggregated: Multi-Objective (MO) Samples

A weighted sample of size $k = \epsilon^{-2}$ with respect to f gives estimates of $Q(g, H)$ with CV $\leq \epsilon\sqrt{\rho/q}$.
$\implies$ guarantees on quality for $Q(g, H)$ degrades with disparity $\rho(f, g)$.

What if we want CV $\leq \epsilon/\sqrt{q}$ for several $f \in F$?

Aggregated: Multi-Objective (MO) Samples

A weighted sample of size $k = \epsilon^{-2}$ with respect to f gives estimates of $Q(g, H)$ with CV $\leq \epsilon\sqrt{\rho/q}$.
$\Longrightarrow$ guarantees on quality for $Q(g, H)$ degrades with disparity $\rho(f, g)$.

What if we want CV $\leq \epsilon/\sqrt{q}$ for several $f \in F$?
Naive solution: Use $|F|$ independent samples S_f for $f \in F$. Size is $|F|\epsilon^{-2}$.

Aggregated: Multi-Objective (MO) Samples

A weighted sample of size $k = \epsilon^{-2}$ with respect to f gives estimates of $Q(g, H)$ with CV $\leq \epsilon\sqrt{\rho/q}$.
$\implies$ guarantees on quality for $Q(g, H)$ degrades with disparity $\rho(f, g)$.

What if we want CV $\leq \epsilon/\sqrt{q}$ for several $f \in F$?
Naive solution: Use $|F|$ independent samples S_f for $f \in F$. Size is $|F|\epsilon^{-2}$.
Can we do better ?

Multi-objective sample S_F [CKS09]

Aggregated: Multi-Objective (MO) Samples

A weighted sample of size $k = \epsilon^{-2}$ with respect to f gives estimates of $Q(g, H)$ with CV $\leq \epsilon\sqrt{\rho/q}$.
$\implies$ guarantees on quality for $Q(g, H)$ degrades with disparity $\rho(f, g)$.

What if we want CV $\leq \epsilon/\sqrt{q}$ for several $f \in F$?
Naive solution: Use $|F|$ independent samples S_f for $f \in F$. Size is $|F|\epsilon^{-2}$.
Can we do better ?

Multi-objective sample S_F [CKS09]

- $S_F = \bigcup_{f\in F} S_f$ is the union of *coordinated* bottom-k (or pps) samples for $f \in F$: E.g. with priority sampling, draw $u_x \sim U[0, 1]$ once, and for S_f use $\texttt{seed}(x) = u_x/f(w_x)$.
  Coordination [BEJ72, Coh97] makes similar samples S_f for similar f.

Aggregated: Multi-Objective (MO) Samples

A weighted sample of size $k = \epsilon^{-2}$ with respect to f gives estimates of $Q(g, H)$ with CV $\leq \epsilon\sqrt{\rho/q}$.
$\implies$ guarantees on quality for $Q(g, H)$ degrades with disparity $\rho(f, g)$.

What if we want CV $\leq \epsilon/\sqrt{q}$ for several $f \in F$?
Naive solution: Use $|F|$ independent samples S_f for $f \in F$. Size is $|F|\epsilon^{-2}$.
Can we do better ?

Multi-objective sample S_F [CKS09]

- $S_F = \bigcup_{f \in F} S_f$ is the union of *coordinated* bottom-k (or pps) samples for $f \in F$: E.g. with priority sampling, draw $u_x \sim U[0, 1]$ once, and for S_f use $\texttt{seed}(x) = u_x/f(w_x)$.
  Coordination [BEJ72, Coh97] makes similar samples S_f for similar f.
- For estimation, use $p_x = \Pr[x \in S_F]$ (inclusion in at least one S_f)

Aggregated: Multi-Objective (MO) Samples

A weighted sample of size $k = \epsilon^{-2}$ with respect to f gives estimates of $Q(g, H)$ with CV $\leq \epsilon\sqrt{\rho/q}$.
$\implies$ guarantees on quality for $Q(g, H)$ degrades with disparity $\rho(f, g)$.

What if we want CV $\leq \epsilon/\sqrt{q}$ for several $f \in F$?
Naive solution: Use $|F|$ independent samples S_f for $f \in F$. Size is $|F|\epsilon^{-2}$.
Can we do better ?

Multi-objective sample S_F [CKS09]

- $S_F = \bigcup_{f \in F} S_f$ is the union of *coordinated* bottom-k (or pps) samples for $f \in F$: E.g. with priority sampling, draw $u_x \sim U[0, 1]$ once, and for S_f use $\texttt{seed}(x) = u_x/f(w_x)$.
  Coordination [BEJ72, Coh97] makes similar samples S_f for similar f.
- For estimation, use $p_x = \Pr[x \in S_F]$ (inclusion in at least one S_f)

- Estimates have CV $\leq \epsilon/\sqrt{q}$ for $Q(f, H)$ for all $f \in F$.
- Size *typically* $\ll |F|\epsilon^{-2}$ (but is as small as possible).

Aggregated data: MO Sample for all monotone functions

What can we say about MO sampling the set M of all monotone non-decreasing functions of w_x ?

Aggregated data: MO Sample for all monotone functions

What can we say about MO sampling the set M of all monotone non-decreasing functions of w_x ?

M includes all moment, capping, and threshold functions ...

Aggregated data: MO Sample for all monotone functions

What can we say about MO sampling the set M of all monotone non-decreasing functions of w_x ?

M includes all moment, capping, and threshold functions ...

Theorem [Coh15b]

Aggregated data: MO Sample for all monotone functions

What can we say about MO sampling the set M of all monotone non-decreasing functions of w_x ?

M includes all moment, capping, and threshold functions ...

Theorem [Coh15b]

- Size: $\mathrm{E}[|S_M|] \leq \epsilon^{-2} \ln n$, where n number of keys.

Aggregated data: MO Sample for all monotone functions

What can we say about MO sampling the set M of all monotone non-decreasing functions of w_x ?

M includes all moment, capping, and threshold functions ...

Theorem [Coh15b]

- Size: $\mathrm{E}[|S_M|] \leq \epsilon^{-2} \ln n$, where n number of keys.
- Computation: S_M and inclusion probabilities used for estimation can be computed using $O(n \log \epsilon^{-1})$ operations.

Aggregated data: MO Sample for all monotone functions

What can we say about MO sampling the set M of all monotone non-decreasing functions of w_x ?

M includes all moment, capping, and threshold functions ...

Theorem [Coh15b]

- Size: $\mathrm{E}[|S_M|] \leq \epsilon^{-2} \ln n$, where n number of keys.
- Computation: S_M and inclusion probabilities used for estimation can be computed using $O(n \log \epsilon^{-1})$ operations.
- Tight lower bound: When keys have distinct weights, any sample providing these statistical guarantees has size $\Omega(\epsilon^{-2} \ln n)$. Enough to look at thresh functions ($\text{thresh}_T(x) = 1$ if $x \geq T$ and 0 otherwise)

Aggregated data: MO Sample for all monotone functions

What can we say about MO sampling the set M of all monotone non-decreasing functions of w_x ?
M includes all moment, capping, and threshold functions ...

Theorem [Coh15b]

- Size: $\mathrm{E}[|S_M|] \leq \epsilon^{-2} \ln n$, where n number of keys.
- Computation: S_M and inclusion probabilities used for estimation can be computed using $O(n \log \epsilon^{-1})$ operations.
- Tight lower bound: When keys have distinct weights, any sample providing these statistical guarantees has size $\Omega(\epsilon^{-2} \ln n)$. Enough to look at thresh functions ($\text{thresh}_T(x) = 1$ if $x \geq T$ and 0 otherwise)

Sampling scheme builds on a surprising relation to computing All-Distances sketches [Coh97, Coh15a])

Summary: Aggregated data "gold standard" sampling

$\forall f() \geq 0$, with a weighted sample of size k with respect to $f(w_x)$:

- $\forall$ segment H: $\hat{Q}(f, H)$ has CV $\leq \sqrt{\frac{1}{q(f,H)k}}$.
- $\forall\ g() \geq 0,\ H$: $\hat{Q}(g, H)$ has CV $\leq \sqrt{\frac{\rho(g,f)}{q(g,H)k}}$

The Gold Standard

Summary: Aggregated data "gold standard" sampling

$\forall f() \geq 0$, with a weighted sample of size k with respect to $f(w_x)$:

- $\forall$ segment H: $\hat{Q}(f, H)$ has CV $\leq \sqrt{\frac{1}{q(f,H)k}}$.
- $\forall\ g() \geq 0$, H: $\hat{Q}(g, H)$ has CV $\leq \sqrt{\frac{\rho(g,f)}{q(g,H)k}}$

With a multi-objective sample of size $\leq k \ln n$:

$\forall$ monotone $f \geq 0$, segment H: $\hat{Q}(f, H)$ has CV $\leq \sqrt{\frac{1}{q(f,H)k}}$.

Summary: Aggregated data "gold standard" sampling

$\forall f() \geq 0$, with a weighted sample of size k with respect to $f(w_x)$:

- $\forall$ segment H: $\hat{Q}(f,H)$ has CV $\leq \sqrt{\frac{1}{q(f,H)k}}$.
- $\forall\ g() \geq 0$, H: $\hat{Q}(g,H)$ has CV $\leq \sqrt{\frac{\rho(g,f)}{q(g,H)k}}$

With a multi-objective sample of size $\leq k \ln n$:

$\forall$ monotone $f \geq 0$, segment H: $\hat{Q}(f,H)$ has CV $\leq \sqrt{\frac{1}{q(f,H)k}}$.

More properties of sampling aggregated data:

- Quality: Estimates are concentrated
- Computation: Streamed/distributed sampling with state $\propto$ sample size k ! (Samples are composable)
- Applies to unaggregated data with $w_x \equiv \max_{\text{elements } (x,w)} w$

Summary: Aggregated data "gold standard" sampling

$\forall f() \geq 0$, with a weighted sample of size k with respect to $f(w_x)$:

- $\forall$ segment H: $\hat{Q}(f,H)$ has CV $\leq \sqrt{\frac{1}{q(f,H)k}}$.
- $\forall\ g() \geq 0$, H: $\hat{Q}(g,H)$ has CV $\leq \sqrt{\frac{\rho(g,f)}{q(g,H)k}}$

With a multi-objective sample of size $\leq k \ln n$:

$\forall$ monotone $f \geq 0$, segment H: $\hat{Q}(f,H)$ has CV $\leq \sqrt{\frac{1}{q(f,H)k}}$.

More properties of sampling aggregated data:

- Quality: Estimates are concentrated
- Computation: Streamed/distributed sampling with state $\propto$ sample size k ! (Samples are composable)
- Applies to unaggregated data with $w_x \equiv \max_{\text{elements } (x,w)} w$

Desirables with unaggregated data (and $w_x \equiv \sum_{\text{elements } (x,w)} w$):

- Computation: One or two passes, state $\propto k$ (no aggregated view!)
- Quality: Sample size/estimate quality tradeoff near gold standard.

Toolbox for frequency functions on unaggregated streams

- Deterministic algorithms: Misra Gries: [MG82] Space saving [MAEA05] for heavy hitters

Toolbox for frequency functions on unaggregated streams

- Deterministic algorithms: Misra Gries: [MG82] Space saving [MAEA05] for heavy hitters
- Random linear projections (linear sketches): Project vector of key values to a vector with logarithmic dimension. JL transform [JL84] and stable distributions [Ind01] for frequency moments $p \in [0, 2]$.

Toolbox for frequency functions on unaggregated streams

- Deterministic algorithms: Misra Gries: [MG82] Space saving [MAEA05] for heavy hitters
- Random linear projections (linear sketches): Project vector of key values to a vector with logarithmic dimension. JL transform [JL84] and stable distributions [Ind01] for frequency moments $p \in [0, 2]$.
- Sampling-based : Distinct Reservoir Sampling [Knu68] and MinHash sketches [FM85, Coh97] (distinct statistics), Sample and Hold [GM98, EV02, CDK$^+$14] (sum statistics)

Toolbox for frequency functions on unaggregated streams

- Deterministic algorithms: Misra Gries: [MG82] Space saving [MAEA05] for heavy hitters
- Random linear projections (linear sketches): Project vector of key values to a vector with logarithmic dimension. JL transform [JL84] and stable distributions [Ind01] for frequency moments $p \in [0, 2]$.
- Sampling-based : Distinct Reservoir Sampling [Knu68] and MinHash sketches [FM85, Coh97] (distinct statistics), Sample and Hold [GM98, EV02, CDK+14] (sum statistics)

No previous solutions for general capping statistics.

Sampling framework for unaggregated data [Coh15c]

Unifies classic schemes for distinct or sum statistics, generalizes bottom-k

Sampling framework for unaggregated data [Coh15c]

Unifies classic schemes for distinct or sum statistics, generalizes bottom-k

1. Scores of elements

Scheme is specified by a random mapping `ElementScore(`h`)` of elements $h = (x, w)$ to a numeric score.

Sampling framework for unaggregated data [Coh15c]

Unifies classic schemes for distinct or sum statistics, generalizes bottom-k

1. Scores of elements

Scheme is specified by a random mapping ElementScore(h) of elements $h = (x, w)$ to a numeric score.

Properties of ElementScore: Distribution depends only on x and w. Can be dependent for same key, independent for different keys.

Sampling framework for unaggregated data [Coh15c]

Unifies classic schemes for distinct or sum statistics, generalizes bottom-k

1. Scores of elements

Scheme is specified by a random mapping $\texttt{ElementScore}(h)$ of elements $h = (x, w)$ to a numeric score.

Properties of $\texttt{ElementScore}$: Distribution depends only on x and w. Can be dependent for same key, independent for different keys.

2. Seeds of keys

The *seed* of a key x is the minimum score of all its elements.

$$\texttt{seed}(x) = \min_{h \text{ with key } x} \texttt{ElementScore}(h)$$

Sampling framework for unaggregated data [Coh15c]

Unifies classic schemes for distinct or sum statistics, generalizes bottom-k

1. Scores of elements

Scheme is specified by a random mapping `ElementScore(h)` of elements $h = (x, w)$ to a numeric score.

Properties of `ElementScore`: Distribution depends only on x and w. Can be dependent for same key, independent for different keys.

2. Seeds of keys

The *seed* of a key x is the minimum score of all its elements.

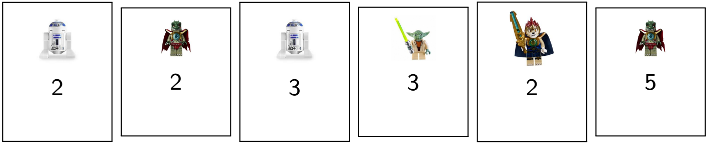
$$\texttt{seed}(x) = \min_{h \text{ with key } x} \texttt{ElementScore}(h)$$

3. Sample (S, τ)

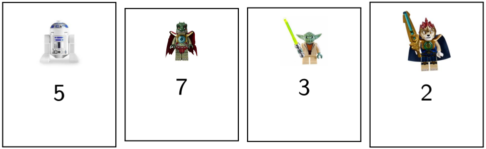
$S \leftarrow$ the k keys with smallest $\texttt{seed}(x)$ (and their seed values)
$\tau \leftarrow$ the $(k+1)$st smallest seed value.

Sampling unaggregated data: Example

Unaggregated data:

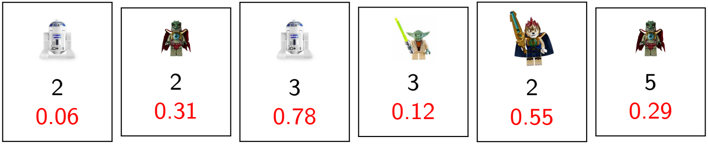


The aggregated view:

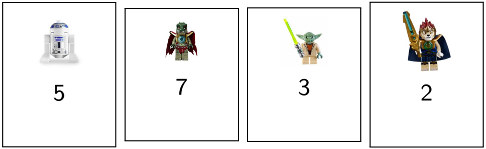


Sampling unaggregated data: Example

Unaggregated data: (with $\texttt{ElementScore}(h)$)



The aggregated view:



Sampling unaggregated data: Example

Unaggregated data: (with $\texttt{ElementScore}(h)$)

2
0.06

2
0.31

3
0.78

3
0.12

2
0.55

5
0.29

The aggregated view:
with $\texttt{seed}(x)$

5
0.06

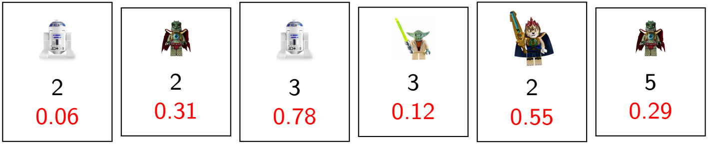
7
0.29

3
0.12

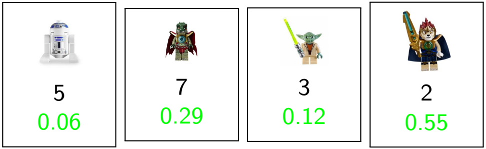
2
0.55

Sampling unaggregated data: Example

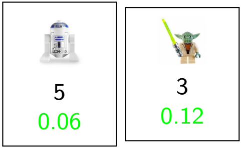
Unaggregated data: (with $\texttt{ElementScore}(h)$)



The aggregated view:
with $\texttt{seed}(x)$



Sample of size $k = 2$:



$\tau = 0.29$

Distinct sampling, casted in our framework

A distinct sample is a uniform sample of k active keys (keys with $w_x > 0$). Reservoir sampling [Knu68] +Hashing [FM85] [Vit85]

Distinct sampling, casted in our framework

A distinct sample is a uniform sample of k active keys (keys with $w_x > 0$). Reservoir sampling [Knu68] +Hashing [FM85] [Vit85]

Scoring for distinct sampling

$$\texttt{ElementScore}(h) = \texttt{Hash}(x), \text{ for random hash } \texttt{Hash}(x) \sim U[0,1]$$

Distinct sampling, casted in our framework

A distinct sample is a uniform sample of k active keys (keys with $w_x > 0$). Reservoir sampling [Knu68] +Hashing [FM85] [Vit85]

Scoring for distinct sampling

$$\texttt{ElementScore}(h) = \texttt{Hash}(x), \text{ for random hash } \texttt{Hash}(x) \sim U[0,1]$$

Correctness: All elements with same key x have the same score and thus $\texttt{seed}(x) \equiv \texttt{Hash}(x)$. The sample is the k active keys with smallest hash.

Distinct sampling, casted in our framework

A distinct sample is a uniform sample of k active keys (keys with $w_x > 0$). Reservoir sampling [Knu68] +Hashing [FM85] [Vit85]

Scoring for distinct sampling

$$\texttt{ElementScore}(h) = \texttt{Hash}(x), \text{ for random hash } \texttt{Hash}(x) \sim U[0,1]$$

Correctness: All elements with same key x have the same score and thus $\texttt{seed}(x) \equiv \texttt{Hash}(x)$. The sample is the k active keys with smallest hash.

From the point x is included in S, we maintain a count c_x of the sum of weights of its elements. Since any key entered the sample on its first element, we have $c_x = w_x$.

Estimation from a distinct sample

Each key x with $w_x > 0$ is sampled (conditioned on hashes of other keys) with probability $p_{x|\tau} \equiv \tau$.

Estimation from a distinct sample

Each key x with $w_x > 0$ is sampled (conditioned on hashes of other keys) with probability $p_{x|\tau} \equiv \tau$.

Since we also know w_x, we can use for any f the unbiased inverse probability estimate [HT52]:

$$\hat{Q}(f, H) = \sum_{x \in S \cap H} \frac{f(w_x)}{p_{x|\tau}} = \frac{1}{\tau} \sum_{x \in S \cap H} f(w_x) \; .$$

Estimation from a distinct sample

Each key x with $w_x > 0$ is sampled (conditioned on hashes of other keys) with probability $p_{x|\tau} \equiv \tau$.

Since we also know w_x, we can use for any f the unbiased inverse probability estimate [HT52]:

$$\hat{Q}(f, H) = \sum_{x \in S \cap H} \frac{f(w_x)}{p_{x|\tau}} = \frac{1}{\tau} \sum_{x \in S \cap H} f(w_x) \ .$$

Estimate quality: The sample and estimator are ppswor with respect to weights $f(w) \equiv \text{cap}_1$

$\Longrightarrow$ For a segment H with proportion q, $\hat{Q}(\text{cap}_1, H)$ has CV $\approx \sqrt{\frac{1}{qk}}$.

Estimation from a distinct sample

Each key x with $w_x > 0$ is sampled (conditioned on hashes of other keys) with probability $p_{x|\tau} \equiv \tau$.

Since we also know w_x, we can use for any f the unbiased inverse probability estimate [HT52]:

$$\hat{Q}(f, H) = \sum_{x \in S \cap H} \frac{f(w_x)}{p_{x|\tau}} = \frac{1}{\tau} \sum_{x \in S \cap H} f(w_x) \; .$$

Estimate quality: The sample and estimator are ppswor with respect to weights $f(w) \equiv \text{cap}_1$

$\Longrightarrow$ For a segment H with proportion q, $\hat{Q}(\text{cap}_1, H)$ has CV $\approx \sqrt{\frac{1}{qk}}$.

$\Longrightarrow$ For cap_T statistics, disparity is $\rho(\text{cap}_1, \text{cap}_T) = T$. The bound on the CV of $\hat{Q}(\text{cap}_T, H)$ is $\sqrt{\frac{T}{qk}}$. Intuitively, our sample can easily miss "heavy" keys with high $\text{cap}_T(w_x)$ values which contribute more to the statistics.

Sampling for sum statistics

Sample and Hold (counting samples) [GM98, EV02]:

If $x \in S$, increment c_x. Otherwise, cache if $\texttt{rand()} < \tau$.

Can be used with a fixed-size sample k; Equivalent to ppswor [CDK+14]; Continuous version (element weights) [CCD11].

Sampling for sum statistics

Sample and Hold (counting samples) [GM98, EV02]:

If $x \in S$, increment c_x. Otherwise, cache if $\texttt{rand()} < \tau$.

Can be used with a fixed-size sample k; Equivalent to ppswor [CDK$^+$14]; Continuous version (element weights) [CCD11].

Sample and Hold casted in our framework:

Element scoring function

$$\texttt{ElementScore}(h{=}(x,w)) \sim \text{Exp}[w]$$

Sampling for sum statistics

Sample and Hold (counting samples) [GM98, EV02]:

If $x \in S$, increment c_x. Otherwise, cache if $\texttt{rand()} < \tau$.

Can be used with a fixed-size sample k; Equivalent to ppswor [CDK+14]; Continuous version (element weights) [CCD11].

Sample and Hold casted in our framework:

Element scoring function

$$\texttt{ElementScore}(h{=}(x,w)) \sim \text{Exp}[w]$$

The minimum of independent exponential random variables is an exponential random variable with a parameter that is the sum of their parameters. We get

$$\texttt{seed}(x) \sim \min_{\text{elements } (x,w)} \text{Exp}[w] \equiv \text{Exp}[w_x] \implies \text{ppswor wrt } w_x!$$

Unaggregated data: Estimating sum statistics from ppswor

Caveat! We do have a ppswor sample S and the threshold τ, but **exact** weights w_x for $x \in S$ are needed for the inverse probability estimator. When streaming (single pass), we can start "counting" w_x only after x enters the cache, so we may miss some elements and only have $c_x < w_x$.

Unaggregated data: Estimating sum statistics from ppswor

Caveat! We do have a ppswor sample S and the threshold τ, but **exact** weights w_x for $x \in S$ are needed for the inverse probability estimator. When streaming (single pass), we can start "counting" w_x only after x enters the cache, so we may miss some elements and only have $c_x < w_x$.

Solutions:

Unaggregated data: Estimating sum statistics from ppswor

Caveat! We do have a ppswor sample S and the threshold τ, but **exact** weights w_x for $x \in S$ are needed for the inverse probability estimator. When streaming (single pass), we can start "counting" w_x only after x enters the cache, so we may miss some elements and only have $c_x < w_x$.

Solutions:

- 2-passes: Use the first pass to identify the set S of sampled keys. Use a second pass to exactly count w_x for sampled keys. Apply ppswor inverse probability estimator.

Unaggregated data: Estimating sum statistics from ppswor

Caveat! We do have a ppswor sample S and the threshold τ, but **exact** weights w_x for $x \in S$ are needed for the inverse probability estimator. When streaming (single pass), we can start "counting" w_x only after x enters the cache, so we may miss some elements and only have $c_x < w_x$.

Solutions:

- 2-passes: Use the first pass to identify the set S of sampled keys. Use a second pass to exactly count w_x for sampled keys. Apply ppswor inverse probability estimator.
- Work with c_x: For estimating sum statistics, we can add expected weight of missed prefix [GM98, EV02, CDK+14] (discrete) [CCD11] (continuous) to each sampled key in segment to obtain an unbiased estimate.

Possible to estimate unbiasedly general f.... [CDK+14] (discrete) [Coh15c] (continuous)... more later.

ℓ-capped sampling [Coh15c]

Hurdle 1

To obtain a sample with gold standard quality for cap_ℓ, we need element scoring that would result in inclusion probability roughly proportional to $\text{cap}_\ell(w_x)$

ℓ-capped sampling [Coh15c]

Hurdle 1

To obtain a sample with gold standard quality for cap_ℓ, we need element scoring that would result in inclusion probability roughly proportional to $\mathrm{cap}_\ell(w_x)$

Hurdle 2

Streaming: Even if we have the "right" sampling probabilities, when using a single pass we need estimators that work with observed counts c_x instead of with w_x

ℓ-capped sampling: Hurdle 1 

Obtaining inclusion probabilities roughly proportional to $\mathrm{cap}_\ell(w_x)$

Each key has a *base hash* $\texttt{KeyBase}(x) \sim U[0, 1/\ell]$, obtained using $\texttt{KeyBase}(x) \leftarrow \texttt{Hash}(x)/\ell$. An element $h = (x, w)$ is assigned a score by first drawing $v \sim \mathrm{Exp}[w]$ and then returning v if $v > 1/\ell$ and $\texttt{KeyBase}(x)$ otherwise:

element scoring for ℓ-capped samples

$$\texttt{ElementScore}(h) = (v \sim \mathrm{Exp}[w]) \leq 1/\ell \; ? \; \texttt{KeyBase}(x) : v$$

The $\mathrm{Exp}[w]$ draws are independent for different elements and independent of $\texttt{KeyBase}(x)$.

ℓ-capped sampling: Hurdle 1 

Obtaining inclusion probabilities roughly proportional to $\text{cap}_\ell(w_x)$

Each key has a *base hash* $\texttt{KeyBase}(x) \sim U[0, 1/\ell]$, obtained using $\texttt{KeyBase}(x) \leftarrow \texttt{Hash}(x)/\ell$. An element $h = (x, w)$ is assigned a score by first drawing $v \sim \text{Exp}[w]$ and then returning v if $v > 1/\ell$ and $\texttt{KeyBase}(x)$ otherwise:

element scoring for ℓ-capped samples

$$\texttt{ElementScore}(h) = (v \sim \text{Exp}[w]) \leq 1/\ell \;?\; \texttt{KeyBase}(x) : v$$

The $\text{Exp}[w]$ draws are independent for different elements and independent of $\texttt{KeyBase}(x)$.

$\texttt{seed}(x)$ distribution

$$\texttt{seed}(x) \sim (v \sim \text{Exp}[w_x]) \leq 1/\ell \;?\; U[0, 1/\ell] : v$$

ℓ-capped sampling: Hurdle 1 

Obtaining inclusion probabilities roughly proportional to $\mathrm{cap}_\ell(w_x)$

Each key has a *base hash* $\texttt{KeyBase}(x) \sim U[0, 1/\ell]$, obtained using $\texttt{KeyBase}(x) \leftarrow \texttt{Hash}(x)/\ell$. An element $h = (x, w)$ is assigned a score by first drawing $v \sim \mathrm{Exp}[w]$ and then returning v if $v > 1/\ell$ and $\texttt{KeyBase}(x)$ otherwise:

element scoring for ℓ-capped samples

$$\texttt{ElementScore}(h) = (v \sim \mathrm{Exp}[w]) \leq 1/\ell \ ?\ \texttt{KeyBase}(x) : v$$

The $\mathrm{Exp}[w]$ draws are independent for different elements and independent of $\texttt{KeyBase}(x)$.

$\texttt{seed}(x)$ distribution

$$\texttt{seed}(x) \sim (v \sim \mathrm{Exp}[w_x]) \leq 1/\ell \ ?\ U[0, 1/\ell] : v$$

- For keys with $w_x \ll \ell$, this is like ppswor wrt w_x

ℓ-capped sampling: Hurdle 1 

Obtaining inclusion probabilities roughly proportional to $\text{cap}_\ell(w_x)$

Each key has a *base hash* $\texttt{KeyBase}(x) \sim U[0, 1/\ell]$, obtained using $\texttt{KeyBase}(x) \leftarrow \texttt{Hash}(x)/\ell$. An element $h = (x, w)$ is assigned a score by first drawing $v \sim \text{Exp}[w]$ and then returning v if $v > 1/\ell$ and $\texttt{KeyBase}(x)$ otherwise:

element scoring for ℓ-capped samples

$$\texttt{ElementScore}(h) = (v \sim \text{Exp}[w]) \leq 1/\ell \; ? \; \texttt{KeyBase}(x) : v$$

The $\text{Exp}[w]$ draws are independent for different elements and independent of $\texttt{KeyBase}(x)$.

$\texttt{seed}(x)$ distribution

$$\texttt{seed}(x) \sim (v \sim \text{Exp}[w_x]) \leq 1/\ell \; ? \; U[0, 1/\ell] : v$$

- For keys with $w_x \ll \ell$, this is like ppswor wrt w_x
- For keys with $w_x \gg \ell$, this is like distinct sampling

2-pass estimation quality

With 2-passes, we have w_x, can compute inclusion probabilities from τ and the distribution, and apply the inverse probability estimator.

Theorem

The CV of estimating $Q(cap_T, H)$ from an ℓ-capped sample of size k with exact weights w_x is at most

$$\left(\frac{e}{e-1}\frac{\max\{T/\ell, \ell/T\}}{q(k-1)}\right)^{0.5} .$$

2-pass estimation quality

With 2-passes, we have w_x, can compute inclusion probabilities from τ and the distribution, and apply the inverse probability estimator.

Theorem

The CV of estimating $Q(cap_T, H)$ from an ℓ-capped sample of size k with exact weights w_x is at most

$$\left(\frac{e}{e-1}\frac{\rho}{q(k-1)}\right)^{0.5} .$$

- $\rho = \max\{T/\ell, \ell/T\}$ is the disparity between cap_ℓ and cap_T.

2-pass estimation quality

With 2-passes, we have w_x, can compute inclusion probabilities from τ and the distribution, and apply the inverse probability estimator.

Theorem

The CV of estimating $Q(cap_T, H)$ from an ℓ-capped sample of size k with exact weights w_x is at most

$$\left(\frac{e}{e-1}\frac{\rho}{q(k-1)}\right)^{0.5} .$$

- $\rho = \max\{T/\ell, \ell/T\}$ is the disparity between cap_ℓ and cap_T.
- Overhead factor of $(\frac{e}{e-1})^{0.5} \approx 1.26$ over aggregated "gold standard."

2-pass estimation quality

With 2-passes, we have w_x, can compute inclusion probabilities from τ and the distribution, and apply the inverse probability estimator.

Theorem

The CV of estimating $Q(cap_T, H)$ from an ℓ-capped sample of size k with exact weights w_x is at most

$$\left(\frac{e}{e-1}\frac{\rho}{q(k-1)}\right)^{0.5}.$$

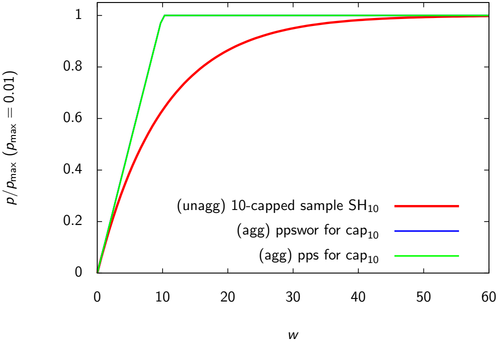
- $\rho = \max\{T/\ell, \ell/T\}$ is the disparity between cap_ℓ and cap_T.
- Overhead factor of $(\frac{e}{e-1})^{0.5} \approx 1.26$ over aggregated "gold standard."
- This is a worst case factor (many items with $w_x = O(\ell)$)

Estimation quality: 2-pass vs. gold standard

10-capped sample, pps and ppswor with weights $\text{cap}_{10}(w)$.

- x axis: the key weight w
- y axis: ratio of inclusion probability to max inclusion probability (set to 0.01).

Ratio gap between curves is maximizes at $w = 10$ and is $(1 - 1/e)$. It is the loss of 10-capped versus aggregated gold standard.



Streaming estimators: Hurdle 2 

The streaming algorithm maintains an "observed count" c_x for $x \in S$:

Streaming estimators: Hurdle 2 

The streaming algorithm maintains an “observed count” c_x for $x \in S$:

- When we process an element $h = (x, w)$ and $x \in S$, we increase $c_x \leftarrow c_x + w$.

Streaming estimators: Hurdle 2 

The streaming algorithm maintains an "observed count" c_x for $x \in S$:

- When we process an element $h = (x, w)$ and $x \in S$, we increase $c_x \leftarrow c_x + w$.
- When the threshold τ decreases, counts c_x are decreased to simulate the result of sampling with respect to the new threshold.

Streaming estimators: Hurdle 2 

The streaming algorithm maintains an "observed count" c_x for $x \in S$:

- When we process an element $h = (x, w)$ and $x \in S$, we increase $c_x \leftarrow c_x + w$.
- When the threshold τ decreases, counts c_x are decreased to simulate the result of sampling with respect to the new threshold.

$\Longrightarrow$ c_x is an r.v. with distribution $\sim D[\tau, \ell, w_x]$.

Streaming estimators: Hurdle 2 

The streaming algorithm maintains an "observed count" c_x for $x \in S$:

- When we process an element $h = (x, w)$ and $x \in S$, we increase $c_x \leftarrow c_x + w$.
- When the threshold τ decreases, counts c_x are decreased to simulate the result of sampling with respect to the new threshold.

$\Longrightarrow$ c_x is an r.v. with distribution $\sim D[\tau, \ell, w_x]$.
Distribution D defines a transform $Y[\tau, \ell]$ from weights w_x to observed counts c_x. Our unbiased estimators are derived by applying f to the inverted transform Y^{-1}:

$$\hat{Q}(f, H) = \sum_{x \in H \cap S} \beta^{(f,\tau,\ell)}(c_x) \ .$$

Streaming estimators: Hurdle 2 

The streaming algorithm maintains an "observed count" c_x for $x \in S$:

- When we process an element $h = (x, w)$ and $x \in S$, we increase $c_x \leftarrow c_x + w$.
- When the threshold τ decreases, counts c_x are decreased to simulate the result of sampling with respect to the new threshold.

$\implies$ c_x is an r.v. with distribution $\sim D[\tau, \ell, w_x]$.
Distribution D defines a transform $Y[\tau, \ell]$ from weights w_x to observed counts c_x. Our unbiased estimators are derived by applying f to the inverted transform Y^{-1}:

$$\hat{Q}(f, H) = \sum_{x \in H \cap S} \beta^{(f,\tau,\ell)}(c_x) \; .$$

Where

$$\beta^{(f,\tau,\ell)}(c) \equiv f(c)/\min\{1, \ell\tau\} + f'(c)/\tau$$

Streaming estimators: Hurdle 2 

The streaming algorithm maintains an "observed count" c_x for $x \in S$:

- When we process an element $h = (x, w)$ and $x \in S$, we increase $c_x \leftarrow c_x + w$.
- When the threshold τ decreases, counts c_x are decreased to simulate the result of sampling with respect to the new threshold.

$\implies$ c_x is an r.v. with distribution $\sim D[\tau, \ell, w_x]$.
Distribution D defines a transform $Y[\tau, \ell]$ from weights w_x to observed counts c_x. Our unbiased estimators are derived by applying f to the inverted transform Y^{-1}:

$$\hat{Q}(f, H) = \sum_{x \in H \cap S} \beta^{(f,\tau,\ell)}(c_x) \; .$$

Where

$$\beta^{(f,\tau,\ell)}(c) \equiv f(c)/\min\{1, \ell\tau\} + f'(c)/\tau$$

$*$ Applies when f is continuous and differentiable almost everywhere (this includes all monotone functions)

Streaming estimator quality

Theorem

The CV of the streaming estimator $\hat{Q}(cap_T, H)$ applied to an ℓ-capped sample is upper bounded by

$$\left(\frac{\frac{e}{e-1}(1+\max\{\ell/T, T/\ell\})}{q(k-1)}\right)^{0.5} .$$

Streaming estimator quality

Theorem

The CV of the streaming estimator $\hat{Q}(cap_T, H)$ applied to an ℓ-capped sample is upper bounded by

$$\left(\frac{\frac{e}{e-1}(1+\max\{\ell/T, T/\ell\})}{q(k-1)}\right)^{0.5}.$$

Worst-case overhead over aggregated "gold standard."

(pseudo) Code: Fixed-k 2-pass distributed ℓ-capped sampling

// Pass I: Identify k keys in Sample

```
// Pass I: Thread adds elements to local summary
Sample ← ∅ // Initialize max heap/dict of key seed pairs
foreach element h = (x, w) do
    if x is in Sample then
        Sample[x].seed ← min{Sample[x].seed, ElementScore(h)}
    else
        s ← ElementScore(h)
        if s < max{Sample[x].seed} then
            Initialize Sample[x]
            Sample[x].seed ← s;
            if |Sample| = k + 1 then
                y ← arg max{Sample[x].seed}
                delete Sample[y]

// Pass I: Merge two summaries Sample, Sample2
foreach x ∈ Sample2 do
    if x is in Sample then
        Sample[x].seed ← min{Sample[x].seed, Sample2[x].seed}
    else
        if Sample2[x].seed < max{Sample[x].seed} then
            Initialize Sample[x]
            Sample[x].seed ← Sample2[x].seed;
            if |Sample| = k + 1 then
                y ← arg max{Sample[x].seed}
                delete Sample[y]
```

// Pass II: Compute w_x for keys in Sample

```
// Pass II: Process elements in thread
foreach x ∈ Sample do // Initialize thread
    Sample[x].w ← 0
foreach element h = (x, w) do
    if x ∈ Sample then
        Sample[x].w ← Sample[x] + w

// Pass II: Merge two summaries Sample, Sample2
foreach x ∈ Sample do
    Sample[x].w ← Sample[x].w + Sample2[x].w
```

(pseudo) Code: Fixed-k stream ℓ-capped sampling

```
foreach stream element (x, w) do // Process element
    if x is in Counters then
        Counters[x] ← Counters[x] + w;
    else
        Δ ← − ln(1−rand()) / max{ℓ^−1, τ} // ∼ Exp[max{ℓ^−1, τ}]
        if Δ < w and (τℓ > 1 or τℓ ≤ 1 and KeyBase(x) < τ) then // insert x
            Counters[x] ← w − Δ
            if |Counters| = k + 1 then // Evict a key
                if τℓ > 1 then
                    foreach x ∈ Counters do
                        u_x ← rand(); r_x ← rand() ; z_x ← min{τ u_x, − ln(1−r_x) / Counters[x]}// x's evict threshold
                        if z_x ≤ ℓ^−1 then
                            z_x ← KeyBase(x)
                    y ← arg max_{x∈Counters} z_x; delete y from Counters // key to evict
                    τ* ← z_y// new threshold
                    foreach x ∈ Counters do // Adjust counters according to τ*
                        if u_x > max{τ*, ℓ^−1}/τ then
                            Counters[x] ←− − ln(1−r_x) / max{ℓ^−1, τ*}
                    τ ← τ*; delete u, r, z, b // deallocate memory
                else // τℓ ≤ 1
                    y ← arg max_{x∈Counters} KeyBase(x); Delete y from Counters // evict y
                    τ ← KeyBase(y)// new threshold
return(τ; (x, Counters[x]) for x in Counters)
```

Simulations

CV upper bounds of $\sqrt{\rho\frac{e}{e-1}/(qk)}$ (2-pass) and $\sqrt{\frac{e}{e-1}(1+\rho)/(qk)}$ (1-pass) are worst-case.

What is the behavior on realistic instances ?

- Quantify gain from second pass
- Understand actual dependence on disparity
- How much do we gain from skew (as in aggregated data) ?

Experiments on Zipf distributions:

- Zipf parameters $\alpha \in [1, 2]$
- Segment=full population
- Swept query cap T and sampling-scheme cap ℓ.

Simulation Results for ℓ-capped samples

Zipf with parameter $\alpha = 2$, sample size $k = 50$, $m = 10^5$ elements.
NRMSE (500 reps) of estimating $Q(\text{cap}_T, \mathcal{X})$ from ℓ-capped sample.

1-pass: $k = 50$, $\alpha = 2$, $m = 100000$, $rep = 500$, NRMSE

ℓ, T	1	5	20	50	100	500	1000	10000
0.1	**0.126**	0.159	0.216	0.274	0.326	0.502	0.597	1.061
1	0.129	0.141	0.192	0.244	0.293	0.449	0.526	0.908
5	0.193	**0.138**	0.146	0.173	0.202	0.300	0.353	0.626
20	0.277	0.169	**0.124**	0.118	0.125	0.183	0.216	0.377
50	0.339	0.206	0.140	0.108	0.094	0.096	0.108	0.182
100	0.390	0.236	0.146	**0.107**	0.085	0.046	0.034	0.022
500	0.397	0.250	0.162	0.114	0.092	0.047	0.034	0.012
1000	0.396	0.232	0.150	0.108	**0.083**	**0.042**	**0.031**	**0.011**
10000	0.404	0.244	0.155	0.114	0.085	0.043	0.032	0.012

2-pass: $k = 50$, $\alpha = 2$, $m = 100000$, $rep = 500$, NRMSE

ℓ, T	1	5	20	50	100	500	1000	10000
0.1	**0.125**	0.159	0.216	0.274	0.326	0.502	0.597	1.061
1	0.127	0.139	0.190	0.244	0.293	0.449	0.526	0.908
5	0.178	**0.137**	0.144	0.172	0.202	0.300	0.353	0.626
20	0.235	0.163	**0.123**	0.116	0.125	0.183	0.216	0.378
50	0.282	0.184	0.133	0.106	0.093	0.094	0.106	0.181
100	0.327	0.204	0.140	0.105	0.083	0.041	0.030	0.020
500	0.321	0.218	0.152	0.114	0.089	0.042	0.030	0.010
1000	0.322	0.208	0.143	**0.105**	**0.080**	**0.039**	**0.028**	**0.009**
10000	0.326	0.213	0.147	0.109	0.084	0.040	0.028	0.010

Worst-case: $0.14 \times 1.26 \times \sqrt{\rho} \approx 0.17\sqrt{\rho}$ (2-pass) $0.17 \times \sqrt{1+\rho}$ (1-pass)

Observations from Simulations

- Actual NRMSE is lower than worst-case:
  - We do not see the $\sqrt{e/(e-1)}$ factor (comes in when many keys have $w_x \approx \ell$).
  - Gain from skew: Observed for large T
    - Note that when $T \ll \ell$, skew can hurt us on "worst-case" segments of many light keys

Observations from Simulations

- Actual NRMSE is lower than worst-case:
  - We do not see the $\sqrt{e/(e-1)}$ factor (comes in when many keys have $w_x \approx \ell$).
  - Gain from skew: Observed for large T
    - Note that when $T \ll \ell$, skew can hurt us on "worst-case" segments of many light keys
- Much better to use $\ell \approx T$

Observations from Simulations

- Actual NRMSE is lower than worst-case:
    - We do not see the $\sqrt{e/(e-1)}$ factor (comes in when many keys have $w_x \approx \ell$).
    - Gain from skew: Observed for large T
        - Note that when $T \ll \ell$, skew can hurt us on "worst-case" segments of many light keys
- Much better to use $\ell \approx T$
- 2-pass estimation quality is within 10% of 1-pass ($\Longrightarrow$ use 2-pass to distribute computation but not to improve estimation)

Conclusion

Summary:

- Aggregated data: Optimal multi-objective sampling scheme for all monotone f

Conclusion

Summary:

- Aggregated data: Optimal multi-objective sampling scheme for all monotone f
- Unaggregated data: Sampling framework which unifies and extends classic solutions for distinct and sum statistics.
- First solution for mid-range cap_T statistics, nearly matches aggregated gold standard.

Conclusion

Summary:

- Aggregated data: Optimal multi-objective sampling scheme for all monotone f
- Unaggregated data: Sampling framework which unifies and extends classic solutions for distinct and sum statistics.
- First solution for mid-range cap_T statistics, nearly matches aggregated gold standard.

Natural Questions (with partial answers):

- Which other monotone frequency functions can our framework handle, in near "aggregated gold standard" sense?

Conclusion

Summary:

- Aggregated data: Optimal multi-objective sampling scheme for all monotone f
- Unaggregated data: Sampling framework which unifies and extends classic solutions for distinct and sum statistics.
- First solution for mid-range cap_T statistics, nearly matches aggregated gold standard.

Natural Questions (with partial answers):

- Which other monotone frequency functions can our framework handle, in near "aggregated gold standard" sense?
  - Some functions are "hard" for streaming (polynomial lower bounds on state): E.g., moments with $p > 2$ [AMS99], threshold

Conclusion

Summary:

- Aggregated data: Optimal multi-objective sampling scheme for all monotone f
- Unaggregated data: Sampling framework which unifies and extends classic solutions for distinct and sum statistics.
- First solution for mid-range cap_T statistics, nearly matches aggregated gold standard.

Natural Questions (with partial answers):

- Which other monotone frequency functions can our framework handle, in near "aggregated gold standard" sense?
  - Some functions are "hard" for streaming (polynomial lower bounds on state): E.g., moments with $p > 2$ [AMS99], threshold
  - Can handle any f that is a nonnegative combination of cap_T functions: All f such that $f' \leq 1$ and $f'' \leq 0$.

Conclusion

Summary:

- Aggregated data: Optimal multi-objective sampling scheme for all monotone f
- Unaggregated data: Sampling framework which unifies and extends classic solutions for distinct and sum statistics.
- First solution for mid-range cap_T statistics, nearly matches aggregated gold standard.

Natural Questions (with partial answers):

- Which other monotone frequency functions can our framework handle, in near "aggregated gold standard" sense?
  - Some functions are "hard" for streaming (polynomial lower bounds on state): E.g., moments with $p > 2$ [AMS99], threshold
  - Can handle any f that is a nonnegative combination of cap_T functions: All f such that $f' \leq 1$ and $f'' \leq 0$.
  - Can also obtain a multi-objective sample for these functions (logarithmic factor on sample size)

Conclusion

Summary:

- Aggregated data: Optimal multi-objective sampling scheme for all monotone f
- Unaggregated data: Sampling framework which unifies and extends classic solutions for distinct and sum statistics.
- First solution for mid-range cap_T statistics, nearly matches aggregated gold standard.

Natural Questions (with partial answers):

- Which other monotone frequency functions can our framework handle, in near "aggregated gold standard" sense?
  - Some functions are "hard" for streaming (polynomial lower bounds on state): E.g., moments with $p > 2$ [AMS99], threshold
  - Can handle any f that is a nonnegative combination of cap_T functions: All f such that $f' \leq 1$ and $f'' \leq 0$.
  - Can also obtain a multi-objective sample for these functions (logarithmic factor on sample size)
  - What about f with super-linear growth? say $p \in (1, 2]$ moments (handled by linear sketches+stable distributions [Ind01, MW10])
  - Can we support signed updates where $f(\max\{0, w\})$? Perhaps build on techniques from [GLH06, CCD12, Coh15c].

Conclusion

Summary:

- Aggregated data: Optimal multi-objective sampling scheme for all monotone f
- Unaggregated data: Sampling framework which unifies and extends classic solutions for distinct and sum statistics.
- First solution for mid-range cap_T statistics, nearly matches aggregated gold standard.

Natural Questions (with partial answers):

- Which other monotone frequency functions can our framework handle, in near "aggregated gold standard" sense?
- Can we do other aggregates of the elements of a given key ?

Conclusion

Summary:

- Aggregated data: Optimal multi-objective sampling scheme for all monotone f
- Unaggregated data: Sampling framework which unifies and extends classic solutions for distinct and sum statistics.
- First solution for mid-range cap_T statistics, nearly matches aggregated gold standard.

Natural Questions (with partial answers):

- Which other monotone frequency functions can our framework handle, in near "aggregated gold standard" sense?
- Can we do other aggregates of the elements of a given key ?
  - (functions of) Sum: here
  - (functions of) max: small extension to aggregated sampling (through sample coordination)
  - what other aggregations are interesting and can be handled ?

Conclusion

Summary:

- Aggregated data: Optimal multi-objective sampling scheme for all monotone f
- Unaggregated data: Sampling framework which unifies and extends classic solutions for distinct and sum statistics.
- First solution for mid-range cap_T statistics, nearly matches aggregated gold standard.

Natural Questions (with partial answers):

- Which other monotone frequency functions can our framework handle, in near "aggregated gold standard" sense?
- Can we do other aggregates of the elements of a given key ?
- If we only want $Q(\text{cap}_T, \mathcal{X})$, can we do better ?
  - Is there a "Hyperloglog like" [FFGM07] algorithm with sketch size $O(\epsilon^{-2} + \log\log n)$ (instead of $O(\epsilon^{-2}\log n)$) ?
  - Can we use HIP estimators? [Coh15a, Tin14]

Thank you!

Bibliography I

N. Alon, Y. Matias, and M. Szegedy.
The space complexity of approximating the frequency moments.
J. Comput. System Sci., 58:137–147, 1999.

K. R. W. Brewer, L. J. Early, and S. F. Joyce.
Selecting several samples from a single population.
Australian Journal of Statistics, 14(3):231–239, 1972.

E. Cohen, G. Cormode, and N. Duffield.
Structure-aware sampling: Flexible and accurate summarization.
Proceedings of the VLDB Endowment, 2011.

E. Cohen, G. Cormode, and N. Duffield.
Don't let the negatives bring you down: Sampling from streams of signed updates.
In *Proc. ACM SIGMETRICS/Performance*, 2012.

E. Cohen, N. Duffield, H. Kaplan, C. Lund, and M. Thorup.
Algorithms and estimators for accurate summarization of unaggregated data streams.
J. Comput. System Sci., 80, 2014.

E. Cohen, N. Duffield, C. Lund, M. Thorup, and H. Kaplan.
Efficient stream sampling for variance-optimal estimation of subset sums.
SIAM J. Comput., 40(5), 2011.

M. T. Chao.
A general purpose unequal probability sampling plan.
Biometrika, 69(3):653–656, 1982.

Bibliography II

E. Cohen and H. Kaplan.
Summarizing data using bottom-k sketches.
In *ACM PODC*, 2007.

E. Cohen, H. Kaplan, and S. Sen.
Coordinated weighted sampling for estimating aggregates over multiple weight assignments.
VLDB, 2(1–2), 2009.
full: http://arxiv.org/abs/0906.4560.

E. Cohen.
Size-estimation framework with applications to transitive closure and reachability.
J. Comput. System Sci., 55:441–453, 1997.

E. Cohen.
All-distances sketches, revisited: HIP estimators for massive graphs analysis.
TKDE, 2015.

E. Cohen.
Multi-objective weighted sampling.
In *HotWeb*. IEEE, 2015.
full version: http://arxiv.org/abs/1509.07445.

E. Cohen.
Stream sampling for frequency cap statistics.
In *KDD*. ACM, 2015.
full version: http://arxiv.org/abs/1502.05955.

Bibliography III

N. Duffield, M. Thorup, and C. Lund.
Priority sampling for estimating arbitrary subset sums.
J. Assoc. Comput. Mach., 54(6), 2007.

C. Estan and G. Varghese.
New directions in traffic measurement and accounting.
In *SIGCOMM*. ACM, 2002.

P. Flajolet, E. Fusy, O. Gandouet, and F. Meunier.
Hyperloglog: The analysis of a near-optimal cardinality estimation algorithm.
In *Analysis of Algorithms (AofA)*. DMTCS, 2007.

P. Flajolet and G. N. Martin.
Probabilistic counting algorithms for data base applications.
J. Comput. System Sci., 31:182–209, 1985.

R. Gemulla, W. Lehner, and P. J. Haas.
A dip in the reservoir: Maintaining sample synopses of evolving datasets.
In *VLDB*, 2006.

P. Gibbons and Y. Matias.
New sampling-based summary statistics for improving approximate query answers.
In *SIGMOD*. ACM, 1998.

D. G. Horvitz and D. J. Thompson.
A generalization of sampling without replacement from a finite universe.
Journal of the American Statistical Association, 47(260):663–685, 1952.

Bibliography IV

P. Indyk.
Stable distributions, pseudorandom generators, embeddings and data stream computation.
In *Proc. 41st IEEE Annual Symposium on Foundations of Computer Science*, pages 189–197. IEEE, 2001.

W. Johnson and J. Lindenstrauss.
Extensions of Lipschitz mappings into a Hilbert space.
Contemporary Math., 26, 1984.

D. E. Knuth.
The Art of Computer Programming, Vol 2, Seminumerical Algorithms.
Addison-Wesley, 1st edition, 1968.

A. Metwally, D. Agrawal, and A. El Abbadi.
Efficient computation of frequent and top-k elements in data streams.
In *ICDT*, 2005.

J. Misra and D. Gries.
Finding repeated elements.
Technical report, Cornell University, 1982.

M. Monemizadeh and D. P. Woodruff.
1-pass relative-error l_p-sampling with applications.
In *Proc. 21st ACM-SIAM Symposium on Discrete Algorithms*. ACM-SIAM, 2010.

Bibliography V

E. Ohlsson.
Sequential poisson sampling.
J. Official Statistics, 14(2):149–162, 1998.

B. Rosén.
Asymptotic theory for successive sampling with varying probabilities without replacement, I.
The Annals of Mathematical Statistics, 43(2):373–397, 1972.

B. Rosén.
Asymptotic theory for order sampling.
J. Statistical Planning and Inference, 62(2):135–158, 1997.

D. Ting.
Streamed approximate counting of distinct elements: Beating optimal batch methods.
In *KDD*. ACM, 2014.

J.S. Vitter.
Random sampling with a reservoir.
ACM Trans. Math. Softw., 11(1):37–57, 1985.