Weighted Sampling for Scalable Analytics of Large Data Sets

Edith Cohen

[1]Google, CA USA

[2]School of Computer Science
Tel Aviv University, Israel

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Data Model

Key value pairs (x, w_x) (users/activity, IP flows/sizes)

"Aggregated presentation:" Data elements (x, w_x) have unique keys

5

7

3

2

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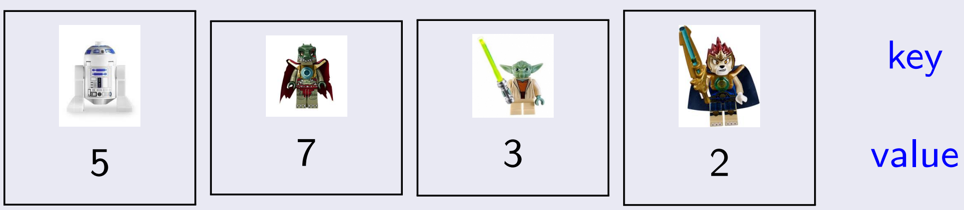

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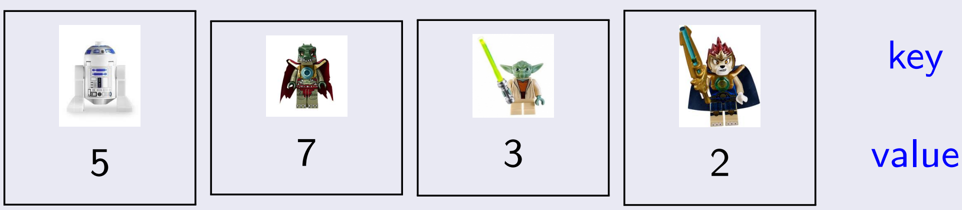

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key

value

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Queries are typically specified over the aggregated view

Summary statistics

$$Q(f, H) = \sum_{x \in H} f(w_x)$$

- Function $f(w) \geq 0$ for $w \geq 0$ so that $f(0) = 0$
- Selected *segment* $H \subset \mathcal{X}$ (domain, subpopulation) from all keys

Example $f()$:

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Moments w^p with $p \in [0, 1]$ and cap statistics cap_T with $T \in (0, +\infty)$ parametrize the range between distinct and sum.

Example queries $Q(f, H)$


135


2


9


18


21


4

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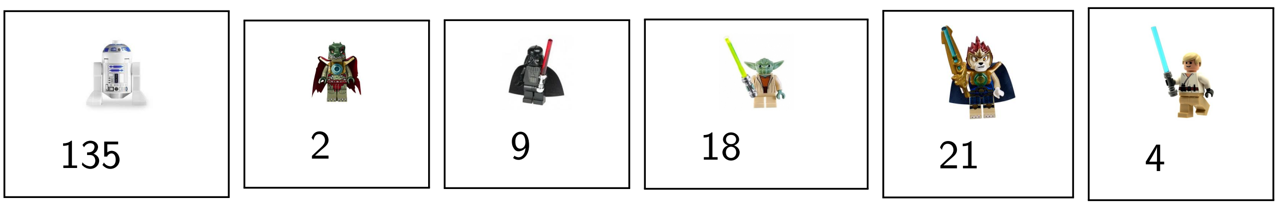

Segment H: space travelers

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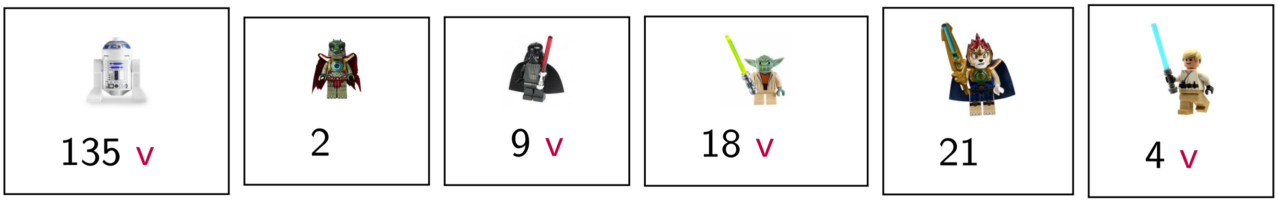

Segment H: space travelers

$Q(count, H) = 4 \qquad Q(\text{cap}_5, H) = 19 \qquad Q(\text{thresh}_{10}, H) = 2$

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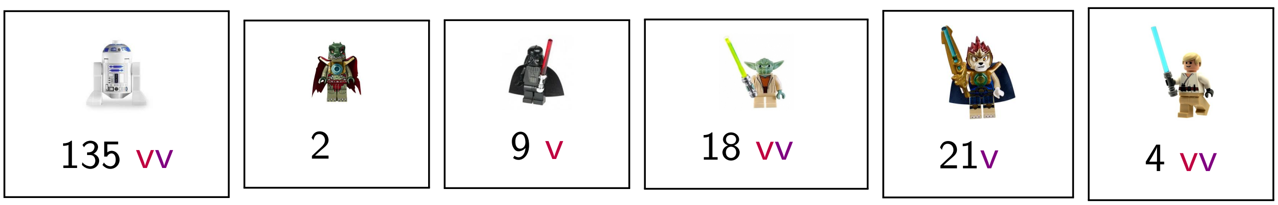

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$Q(count, H) = 3$ $\quad Q(L_2^2, H) = 769$

Challenges

Multi-objective sample (un)aggregated data: For a set of functions F, compute a summary/sample from which we can *estimate* $Q(f, H)$ for various $f \in F$, $H \subseteq \mathcal{X}$.

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Goals:

- Optimize tradeoffs of sample quality (statistical guarantees) and size.
- Scalable computation.

Scalable Computation

One (or few) passes over the data

- Streaming (single sequential pass): Necessary for live dashboards and when data is discarded. Historically model captured sequential-access storage devices (tape, disks), Unix pipes. Streaming model: [Knu68], [MG82], [FM85],. . ., formalized in [AMS99]
- Distributed/Parallel aggregation: Process parts of the data separately and combine small summaries.

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- When streaming, the state is what we keep in memory
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We want state $\ll$ number of (distinct) keys

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Challenge with unaggregated data: Computing the aggregated view $\{(x, w_x)\}$ requires state $\propto$ number of active keys, which can be very large.

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  - Multi-objective (MO) sampling
    - Coordinating samples for different $f \in F$
    - MO sampling scheme for all monotone (non-decreasing) f. [Coh15b]
    - MO sampling distances from query points. [CCK16]

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    - MO sampling distances from query points. [CCK16]
- Unaggregated data sets: How to sample effectively *without* aggregation for capping statistics (and more) [Coh15c]

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Bottom-k/order/weighted reservoir sampling schemes [Ros97, CK07]

foreach *key* x **do** // $Z[w]$: distribution parameterized by w
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- Sequential Poisson (priority) [Ohl98, DTL07]: seed$(x) \sim U[0, 1/f(w_x)]$
- PPS without replacement (ppswor) [Ros72]: seed$(x) \sim \text{Exp}[f(w_x)]$

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Inverse probability estimator of $Q(g, H)$ from the sample S [HT52]

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Poisson PPS samples: $p_x = \min\{1, \frac{kf(w_x)}{\sum_x f(w_x)}\}$
We have w_x for sampled keys $x \in S$, and the total $\sum_x f(w_x)$
$\implies$ can compute p_x and apply estimator.

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The inclusion probability of x *conditioned* on randomization of all other keys: τ is the kth smallest $\mathtt{seed}(y)$ for $y \neq x$; $x \in S \iff \mathtt{seed}(x) < \tau$

- For ppswor $Z[y] \equiv \mathrm{Exp}[y]$: $p_{x|\tau} = 1 - e^{-f(w_x)\tau}$
- For priority $Z[y] \equiv U[0,1/y]$: $p_{x|\tau} = \min\{f(w_x)\tau, 1\}$

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How good is this estimate?

Aggregated: Estimate quality when $g() = f()$

Let $q \equiv q(f, H)$ be the fraction of the statistics f due to segment H:

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+concentration: sample size $k = c\epsilon^{-2}/q$ then prob. of rel. error $> \epsilon$ decreases exponentially in c.

Aggregated: Interpreting the CV bound for $g() = f()$

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!! This is the optimal size/quality tradeoff for sampling (on average over segments with proportion q)


1,567,856


22


89


121


2

For CV $\epsilon \leq 10\%$ and $q \geq 0.1\%$ $\Longrightarrow$ Sample size $k = 10^5$.

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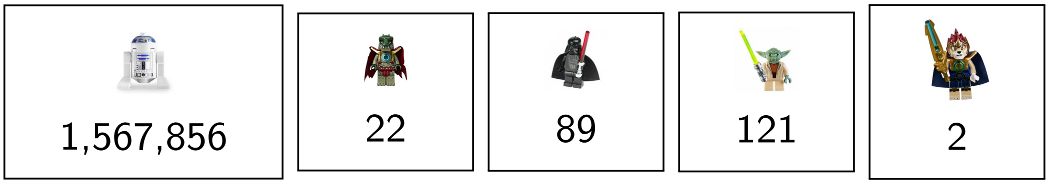

For CV $\epsilon \leq 10\%$ and $q \geq 0.1\%$ $\Longrightarrow$ Sample size $k = 10^5$.

... usually $k \ll$ total number of active keys.

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Lemma

CV of $\hat{Q}(g, H)$ is at most $(\frac{\rho}{q(k-1)})^{0.5}$.

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With a weighted sample of size $k = \epsilon^{-2}$ with respect to f, we estimate $Q(f, H)$ with CV $\leq \epsilon/\sqrt{q}$.
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- Estimates have CV $\leq \epsilon/\sqrt{q}$ for $Q(f, H)$ for all $f \in F$.
- Size *typically* $\ll |F|\epsilon^{-2}$ (but is as small as possible).

Multi-objective Priority (sequential Poisson) sampling

x									
w_x	135	2	9	18	21	4	11	4	2
Count	1	1	1	1	1	1	1	1	1
$\text{cap}_5(w_x)$	5	2	5	5	5	4	5	4	2
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For $k = 3$, the MO sample for $F = \{\mathrm{count}, \mathrm{thresh}_{10}, \mathrm{cap}_5\}$ is:

     

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What can we say about MO sampling the set M of all monotone non-decreasing functions of w_x ?

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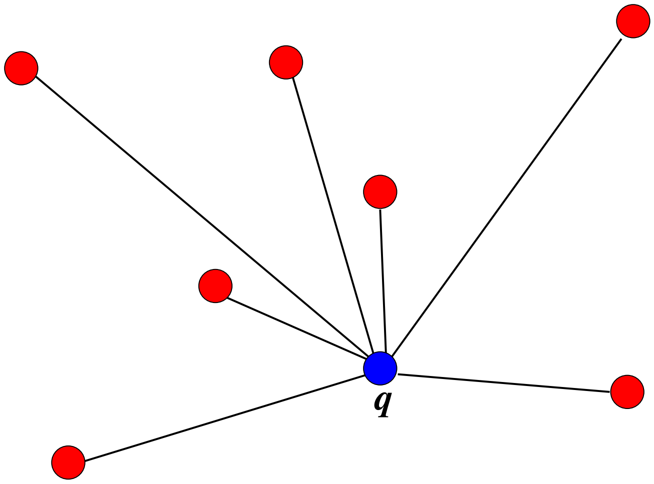
Sampling scheme builds on a surprising relation to computing All-Distances sketches [Coh97, Coh15a])

MO Sample for single-source distances

Set U of points in a metric space M.
Each point q defines $f_q(y) \equiv d_{qy}$, for all $y \in M$.



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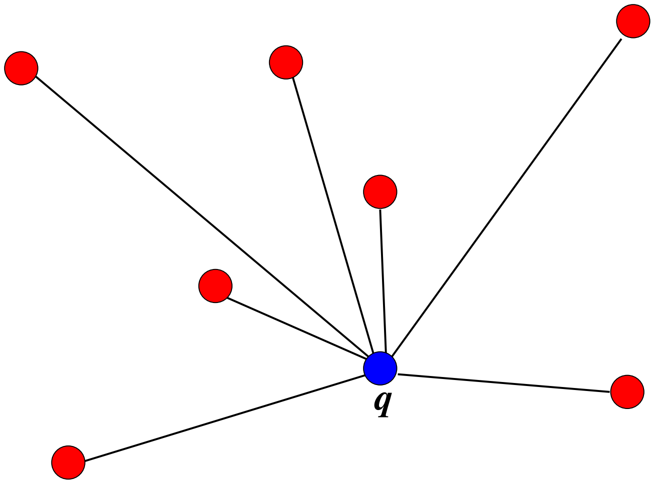

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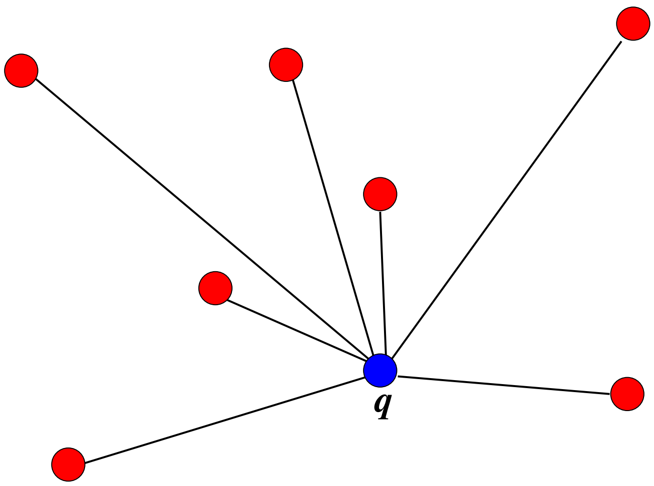

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Theorem

[?] *For any M, $U \subset M$:*

- *The MO sample of f_q for all $q \in M$ has size $O(\epsilon^{-2})$.*
- *The sampling scheme uses $O(|U|)$ pairwise distance computations.*

Talk Overview

- *Aggregated data sets*: Review the "gold standard" Sample size/ estimation quality tradeoffs.
- Multi-objective (MO) sampling
  - Coordinating samples for different $f \in F$
  - MO sampling scheme for all monotone (non-decreasing) f. [Coh15b]
  - MO sampling distances from query points. [CCK16]
- Unaggregated data sets: How to sample effectively *without* aggregation for capping statistics (and more) [Coh15c]

Summary: Aggregated data "gold standard" sampling

$\forall f() \geq 0$, with a weighted sample of size k with respect to $f(w_x)$:

- $\forall$ segment H: $\hat{Q}(f, H)$ has CV $\leq \sqrt{\frac{1}{q(f,H)k}}$.
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Desirables with unaggregated data (and $w_x \equiv \sum_{\text{elements } (x,w)} w$):

- Computation: One or two passes, state $\propto k$ (no aggregated view!)
- Quality: Sample size/estimate quality tradeoff near gold standard.

Application of capping statistics: Online advertising

The first few impressions of the same ad per user are more effective than later ones (diminishing return). Advertisers therefore specify

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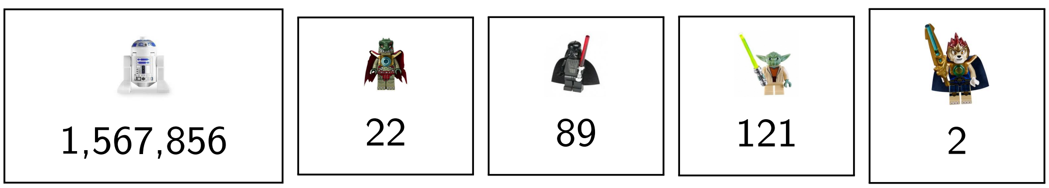

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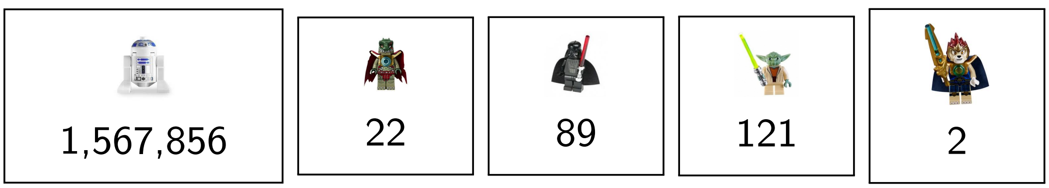

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$\Longrightarrow$ Need quick estimates $\hat{Q}(\text{cap}_T, H)$ from a summary that is computed efficiently over the unaggregated data set.

Toolbox for frequency functions on unaggregated streams

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No effective solutions for general capping statistics.

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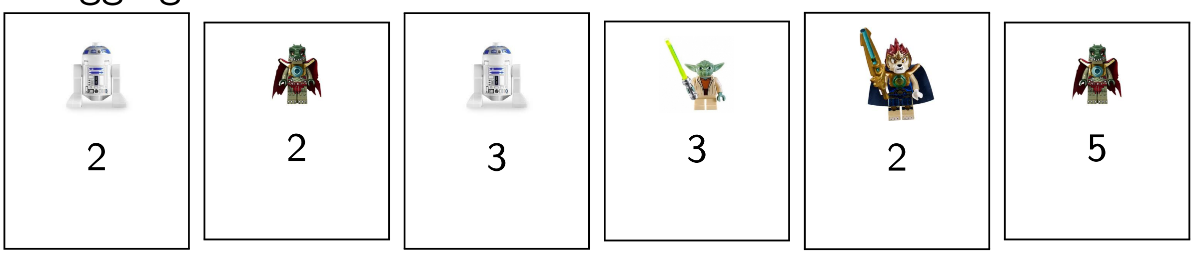
3. Sample (S, τ)

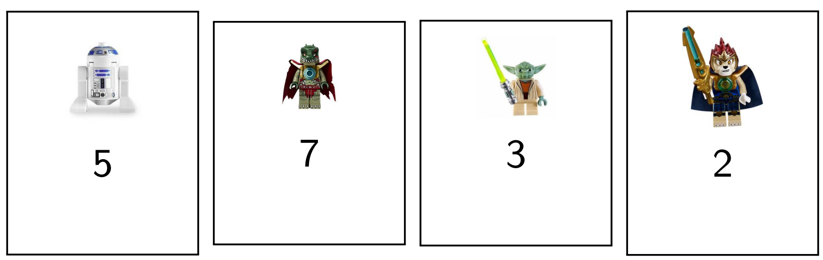
$S \leftarrow$ the k keys with smallest $\texttt{seed}(x)$ (and their seed values)
$\tau \leftarrow$ the $(k+1)$st smallest seed value.

Sampling unaggregated data: Example

Unaggregated data:



The aggregated view:



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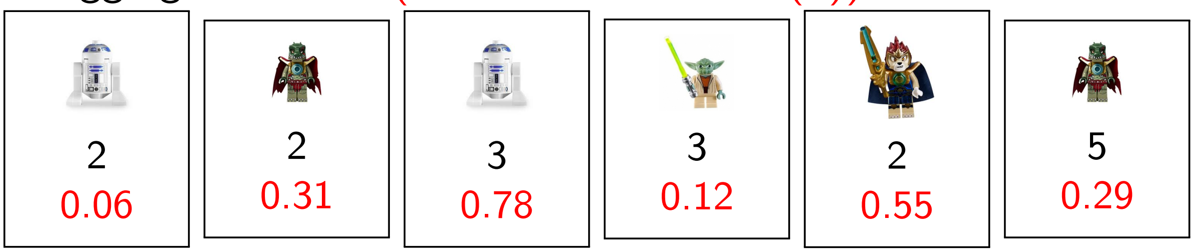

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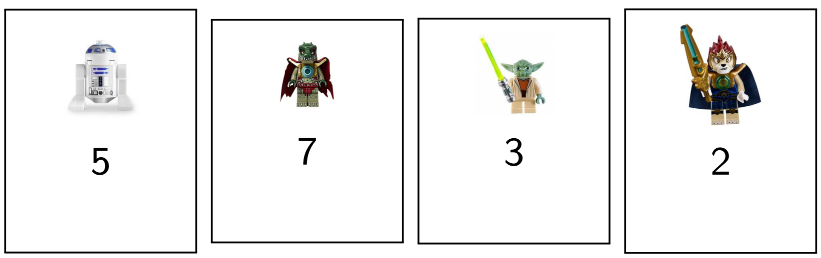

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2
0.06


2
0.31


3
0.78


3
0.12


2
0.55


5
0.29

The aggregated view:
with `seed`(x)


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0.06


7
0.29


3
0.12


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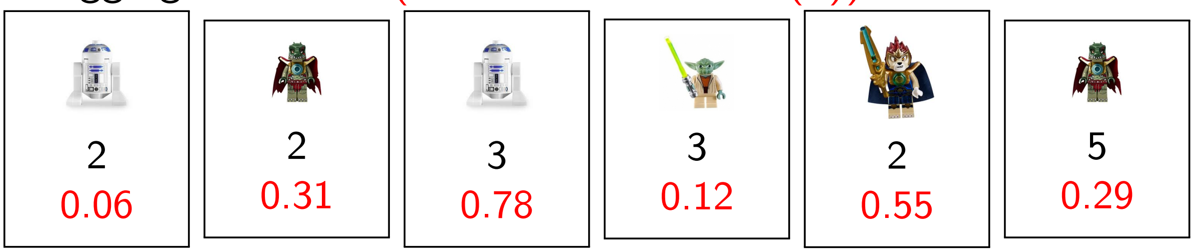

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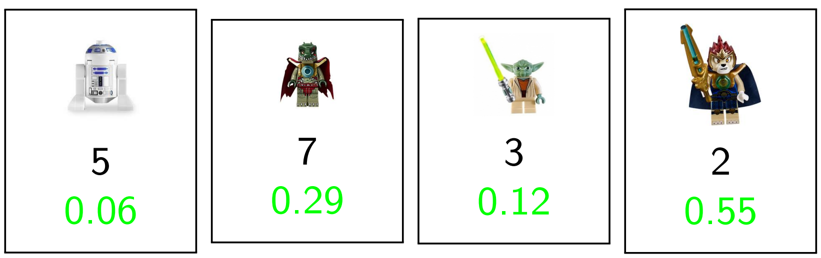
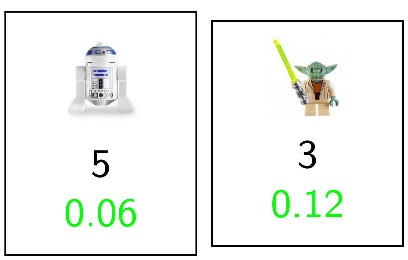


Sample of size $k = 2$:



$\tau = 0.29$

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Correctness: All elements with same key x have the same score and thus $\texttt{seed}(x) \equiv \texttt{Hash}(x)$. The sample is the k active keys with smallest hash.

Distinct sampling, casted in our framework

A distinct sample is a uniform sample of k active keys (keys with $w_x > 0$). Reservoir sampling [Knu68] +Hashing [FM85] [Vit85]

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From the point key x is included in$S$, we maintain a count c_x of the sum of weights of its elements. Since any key entered the sample on its first element, we have $c_x = w_x$.

Estimation from a distinct sample

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$$\hat{Q}(f, H) = \sum_{x \in S \cap H} \frac{f(w_x)}{p_{x|\tau}} = \frac{1}{\tau} \sum_{x \in S \cap H} f(w_x) \; .$$

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Estimate quality: The sample and estimator are ppswor for distinct statistics.

$\Longrightarrow$ For a segment H with proportion q, $\hat{Q}(distinct, H)$ has CV $\approx \sqrt{\frac{1}{qk}}$.

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$\Longrightarrow$ For a segment H with proportion q, $\hat{Q}(\textit{distinct}, H)$ has CV $\approx \sqrt{\frac{1}{qk}}$.

$\Longrightarrow$ For cap_T statistics, disparity is $\rho(\textit{distinct}, \mathrm{cap}_T) = T$. The bound on the CV of $\hat{Q}(\mathrm{cap}_T, H)$ is $\sqrt{\frac{T}{qk}}$. Intuitively, our sample can easily miss "heavy" keys with high $\mathrm{cap}_T(w_x)$ values which contribute more to the statistics.

Sampling for sum statistics

Sample and Hold (counting samples) [GM98, EV02]:

If $x \in S$, increment c_x. Otherwise, cache if $\texttt{rand()} < \tau$.

Can be used with a fixed-size sample k; Equivalent to ppswor [CDK+14]; Continuous version (element weights) [CCD11].

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The minimum of independent exponential random variables is an exponential random variable with a parameter that is the sum of their parameters. We get

$$\texttt{seed}(x) \sim \min_{\text{elements }(x,w)} \text{Exp}[w] \equiv \text{Exp}[w_x] \implies \text{ppswor wrt } w_x!$$

Unaggregated data: Estimating sum statistics from ppswor

Caveat! We do have a ppswor sample S and the threshold τ, but **exact** weights w_x for $x \in S$ are needed for the inverse probability estimator. When streaming (single pass), we can start "counting" w_x only after x enters the cache, so we may miss some elements and only have $c_x < w_x$.

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- 2-passes: Use the first pass to identify the set S of sampled keys. Use a second pass to exactly count w_x for sampled keys. Apply ppswor inverse probability estimator.

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Solutions:

- 2-passes: Use the first pass to identify the set S of sampled keys. Use a second pass to exactly count w_x for sampled keys. Apply ppswor inverse probability estimator.
- Work with c_x: For estimating sum statistics, we can add expected weight of missed prefix [GM98, EV02, CDK+14] (discrete) [CCD11] (continuous) to each sampled key in segment to obtain an unbiased estimate.

Possible to estimate unbiasedly general f.... [CDK+14] (discrete) [Coh15c] (continuous)... more later.

ℓ-capped sampling [Coh15c]

Hurdle 1

To obtain a sample with gold standard quality for cap_ℓ, we need element scoring that would result in inclusion probability roughly proportional to $\text{cap}_\ell(w_x)$

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To obtain a sample with gold standard quality for cap_ℓ, we need element scoring that would result in inclusion probability roughly proportional to $\text{cap}_\ell(w_x)$

Hurdle 2

Streaming: Even if we have the “right” sampling probabilities, when using a single pass we need estimators that work with observed counts c_x instead of with w_x

ℓ-capped sampling: Hurdle 1 

Obtaining inclusion probabilities roughly proportional to $\text{cap}_\ell(w_x)$

Each key has a *base hash* $\texttt{KeyBase}(x) \sim U[0, 1/\ell]$, obtained using $\texttt{KeyBase}(x) \leftarrow \texttt{Hash}(x)/\ell$. An element $h = (x, w)$ is assigned a score by first drawing $v \sim \text{Exp}[w]$ and then returning v if $v > 1/\ell$ and $\texttt{KeyBase}(x)$ otherwise:

element scoring for ℓ-capped samples

$$\texttt{ElementScore}(h) = (v \sim \text{Exp}[w]) \leq 1/\ell \text{ ? } \texttt{KeyBase}(x) : v$$

The $\text{Exp}[w]$ draws are independent for different elements and independent of $\texttt{KeyBase}(x)$.

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- For keys with $w_x \ll \ell$, this is like ppswor wrt w_x
- For keys with $w_x \gg \ell$, this is like distinct sampling

2-pass estimation quality

With 2-passes, we have w_x, can compute inclusion probabilities from τ and the distribution, and apply the inverse probability estimator.

Theorem

The CV of estimating $Q(cap_T, H)$ from an ℓ-capped sample of size k with exact weights w_x is at most

$$\left(\frac{e}{e-1}\frac{\max\{T/\ell, \ell/T\}}{q(k-1)}\right)^{0.5} .$$

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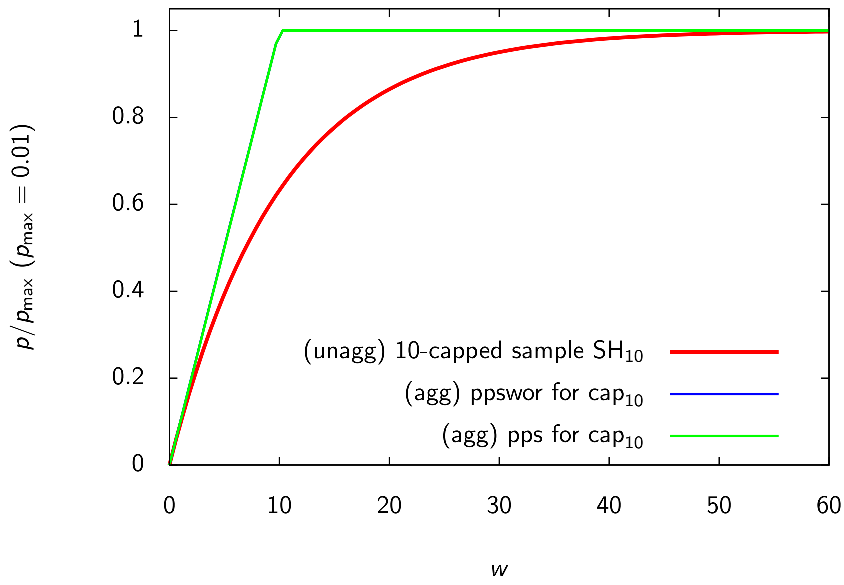
- This is a worst case factor (many items with $w_x = O(\ell)$)

Estimation quality: 2-pass vs. gold standard

10-capped sample, pps and ppswor with weights $\text{cap}_{10}(w)$.

- x axis: the key weight w
- y axis: ratio of inclusion probability to max inclusion probability (set to 0.01).

Ratio gap between curves is maximizes at $w = 10$ and is $(1 - 1/e)$. It is the loss of 10-capped versus aggregated gold standard.



Streaming estimators: Hurdle 2 

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$\implies$ c_x is an r.v. with distribution $\sim D[\tau, \ell, w_x]$.
Distribution D defines a transform $Y[\tau, \ell]$ from weights w_x to observed counts c_x. Our unbiased estimators are derived by applying f to the inverted transform Y^{-1}:

$$\hat{Q}(f, H) = \sum_{x \in H \cap S} \beta^{(f,\tau,\ell)}(c_x) \ .$$

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Where

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$*$ Applies when f is continuous and differentiable almost everywhere (this includes all monotone functions)

Streaming estimator quality

Theorem

The CV of the streaming estimator $\hat{Q}(cap_T, H)$ applied to an ℓ-capped sample is upper bounded by

$$\left(\frac{\frac{e}{e-1}(1+\max\{\ell/T, T/\ell\})}{q(k-1)}\right)^{0.5} .$$

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Worst-case overhead over aggregated "gold standard."

(pseudo) Code: Fixed-k 2-pass distributed ℓ-capped sampling

// Pass I: Identify k keys in Sample

```
// Pass I: Thread adds elements to local summary
Sample ← ∅ // Initialize max heap/dict of key seed pairs
foreach element h = (x, w) do
    if x is in Sample then
        Sample[x].seed ← min{Sample[x].seed, ElementScore(h)}
    else
        s ← ElementScore(h)
        if s < max{Sample[x].seed} then
            Initialize Sample[x]
            Sample[x].seed ← s;
            if |Sample| = k + 1 then
                y ← arg max{Sample[x].seed}
                delete Sample[y]

// Pass I: Merge two summaries Sample, Sample2
foreach x ∈ Sample2 do
    if x is in Sample then
        Sample[x].seed ← min{Sample[x].seed, Sample2[x].seed}
    else
        if Sample2[x].seed < max{Sample[x].seed} then
            Initialize Sample[x]
            Sample[x].seed ← Sample2[x].seed;
            if |Sample| = k + 1 then
                y ← arg max{Sample[x].seed}
                delete Sample[y]
```

// Pass II: Compute w_x for keys in Sample

```
// Pass II: Process elements in thread
foreach x ∈ Sample do // Initialize thread
    Sample[x].w ← 0

foreach element h = (x, w) do
    if x ∈ Sample then
        Sample[x].w ← Sample[x] + w

// Pass II: Merge two summaries Sample, Sample2
foreach x ∈ Sample do
    Sample[x].w ← Sample[x].w + Sample2[x].w
```

(pseudo) Code: Fixed-k stream ℓ-capped sampling

```
foreach stream element (x, w) do // Process element
    if x is in Counters then
        Counters[x] ← Counters[x] + w;
    else
        Δ ← − ln(1−rand()) / max{ℓ^{-1}, τ} // ~ Exp[max{ℓ^{-1}, τ}]
        if Δ < w and (τℓ > 1 or τℓ ≤ 1 and KeyBase(x) < τ) then // insert x
            Counters[x] ← w − Δ
            if |Counters| = k + 1 then // Evict a key
                if τℓ > 1 then
                    foreach x ∈ Counters do
                        u_x ← rand(); r_x ← rand() ; z_x ← min{τ u_x , −ln(1−r_x)/Counters[x]}// x's evict threshold
                        if z_x ≤ ℓ^{-1} then
                            z_x ← KeyBase(x)
                    y ← arg max_{x∈Counters} z_x; delete y from Counters // key to evict
                    τ* ← z_y// new threshold
                    foreach x ∈ Counters do // Adjust counters according to τ*
                        if u_x > max{τ*, ℓ^{-1}}/τ then
                            Counters[x] ← −ln(1−r_x) / max{ℓ^{-1}, τ*}
                    τ ← τ*; delete u, r, z, b // deallocate memory
                else // τℓ ≤ 1
                    y ← arg max_{x∈Counters} KeyBase(x); Delete y from Counters // evict y
                    τ ← KeyBase(y)// new threshold
return(τ; (x, Counters[x]) for x in Counters)
```

Simulations

CV upper bounds of $\sqrt{\rho\frac{e}{e-1}/(qk)}$ (2-pass) and $\sqrt{\frac{e}{e-1}(1+\rho)/(qk)}$ (1-pass) are worst-case.

What is the behavior on realistic instances ?

- Quantify gain from second pass
- Understand actual dependence on disparity
- How much do we gain from skew (as in aggregated data) ?

Experiments on Zipf distributions:

- Zipf parameters $\alpha \in [1, 2]$
- Segment=full population
- Swept query cap T and sampling-scheme cap ℓ.

Simulation Results for ℓ-capped samples

Zipf with parameter $\alpha = 2$, sample size $k = 50$, $m = 10^5$ elements.
NRMSE (500 reps) of estimating $Q(\text{cap}_T, \mathcal{X})$ from ℓ-capped sample.

1-pass: $k = 50,\ \alpha = 2,\ m = 100000,\ rep = 500$, NRMSE

ℓ, T	1	5	20	50	100	500	1000	10000
0.1	**0.126**	0.159	0.216	0.274	0.326	0.502	0.597	1.061
1	0.129	0.141	0.192	0.244	0.293	0.449	0.526	0.908
5	0.193	**0.138**	0.146	0.173	0.202	0.300	0.353	0.626
20	0.277	0.169	**0.124**	0.118	0.125	0.183	0.216	0.377
50	0.339	0.206	0.140	0.108	0.094	0.096	0.108	0.182
100	0.390	0.236	0.146	**0.107**	0.085	0.046	0.034	0.022
500	0.397	0.250	0.162	0.114	0.092	0.047	0.034	0.012
1000	0.396	0.232	0.150	0.108	**0.083**	**0.042**	**0.031**	**0.011**
10000	0.404	0.244	0.155	0.114	0.085	0.043	0.032	0.012

2-pass: $k = 50,\ \alpha = 2,\ m = 100000,\ rep = 500$, NRMSE

ℓ, T	1	5	20	50	100	500	1000	10000
0.1	**0.125**	0.159	0.216	0.274	0.326	0.502	0.597	1.061
1	0.127	0.139	0.190	0.244	0.293	0.449	0.526	0.908
5	0.178	**0.137**	0.144	0.172	0.202	0.300	0.353	0.626
20	0.235	0.163	**0.123**	0.116	0.125	0.183	0.216	0.378
50	0.282	0.184	0.133	0.106	0.093	0.094	0.106	0.181
100	0.327	0.204	0.140	0.105	0.083	0.041	0.030	0.020
500	0.321	0.218	0.152	0.114	0.089	0.042	0.030	0.010
1000	0.322	0.208	0.143	**0.105**	**0.080**	**0.039**	**0.028**	**0.009**
10000	0.326	0.213	0.147	0.109	0.084	0.040	0.028	0.010

Worst-case: $0.14 \times 1.26 \times \sqrt{\rho} \approx 0.17\sqrt{\rho}$ (2-pass) $0.17 \times \sqrt{1+\rho}$ (1-pass)

Observations from Simulations

- Actual NRMSE is lower than worst-case:
  - We do not see the $\sqrt{e/(e-1)}$ factor (comes in when many keys have $w_x \approx \ell$).
  - Gain from skew: Observed for large T
    - Note that when $T \ll \ell$, skew can hurt us on "worst-case" segments of many light keys

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    - Note that when $T \ll \ell$, skew can hurt us on "worst-case" segments of many light keys
- Much better to use $\ell \approx T$
- 2-pass estimation quality is within 10% of 1-pass ($\Longrightarrow$ use 2-pass to distribute computation but not to improve estimation)

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- Which other monotone frequency functions can our framework handle, in near "aggregated gold standard" sense?

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  - Some functions are "hard" for streaming (polynomial lower bounds on state): E.g., moments with $p > 2$ [AMS99], threshold
  - Can handle any f that is a nonnegative combination of cap_T functions: All f such that $f' \leq 1$ and $f'' \leq 0$.

Conclusion

Summary:

- Aggregated data: Optimal multi-objective sampling scheme for all monotone f
- Unaggregated data: Sampling framework which unifies and extends classic solutions for distinct and sum statistics.
- Solution for cap_T statistics, nearly matches aggregated gold standard.

Natural Questions (with partial answers):

- Which other monotone frequency functions can our framework handle, in near "aggregated gold standard" sense?
  - Some functions are "hard" for streaming (polynomial lower bounds on state): E.g., moments with $p > 2$ [AMS99], threshold
  - Can handle any f that is a nonnegative combination of cap_T functions: All f such that $f' \leq 1$ and $f'' \leq 0$.
  - Can also obtain a multi-objective sample for these functions (logarithmic factor on sample size)

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  - Can also obtain a multi-objective sample for these functions (logarithmic factor on sample size)
  - Some f with super-linear growth ($p \in (1, 2]$ moments) is handled by linear sketches [Ind01, MW10] but not by samples.
  - Can we support signed updates where $f(\max\{0, w\})$? Perhaps use [GLH06, CCD12, Coh15c].

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  - (functions of) Sum: here
  - (functions of) max: small extension to aggregated sampling (through sample coordination)
  - what other aggregations are interesting and can be handled ?

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Natural Questions (with partial answers):

- Which other monotone frequency functions can our framework handle, in near "aggregated gold standard" sense?
- Can we do other aggregates of the elements of a given key ?
- If we only want $Q(\text{cap}_T, \mathcal{X})$, can we do better ?
  - Is there a "Hyperloglog like" [FFGM07] algorithm with sketch size $O(\epsilon^{-2} + \log\log n)$ (instead of $O(\epsilon^{-2}\log n)$) ?
  - Can we use HIP estimators? [Coh15a, Tin14]

Thank you!

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