Greedy Maximization Framework for Graph-based Influence Functions

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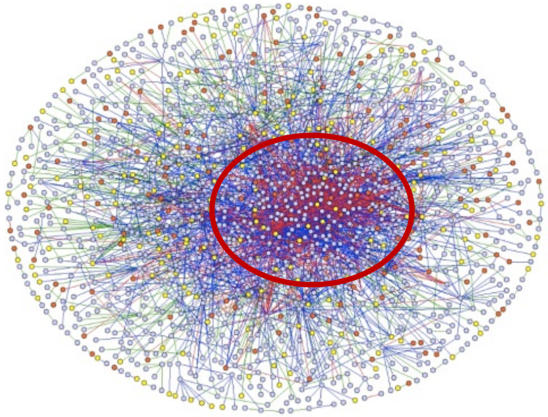
Large Graphs



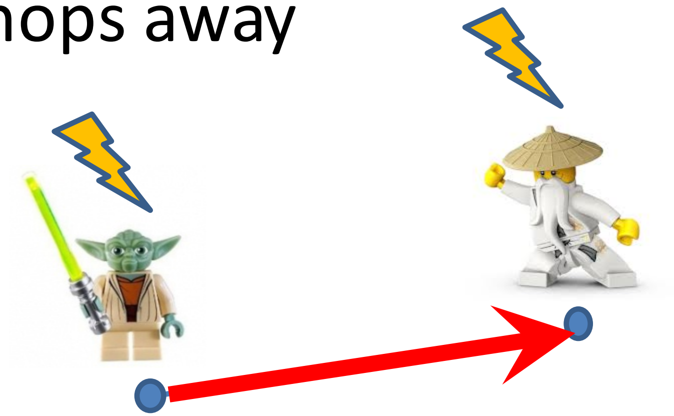
Model relations/interactions (edges) between entities (nodes)

- Explicit: Call detail, email exchanges, Web links, social networks (friend, follow, like), commercial transactions, video views, …
- Implicit: Images, search queries, (edges are shared features or close embedding vectors)

Some nodes are more central than others

Diffusion in Networks

- Edges model *direct connections* between entities
- Diffusion: Contagion, information (news, opinions), … can spread from seed nodes through edges to nodes multiple hops away



- Influence: A measure of the combined power/ importance/ coverage of a set of *seed nodes.* (according to the diffusion process)

Influence in Networks

Applications : $\mathrm{Inf}(S) =$ quality of a seed entities S as:

anchors, representatives, cluster centers, hubs in distribution networks, candidates for active learning of labels/properties, seeds for viral marketing

Sketches

Computational Problems:

- **Influence queries:** Given *seed* set S, compute (approximate) $\mathrm{Inf}(S)$
- **Influence maximization** $\arg\max_{S \mid |S|=s} \mathrm{Inf}(S)$

Find a set of entities with maximum influence for its size.

or With a "budget" s, who should we select ?

Overview of contributions

- A unified model of graph-based influence functions: Includes functions proposed in previous work and extends to allow general submodular aggregations.
- A meta-algorithm for influence maximization: Modular design, near linear computation, statistical worst-case guarantees on approximation quality

Unified model: Pairwise utility to influence

- Graph structure, diffusion process $\Longrightarrow$ pairwise ***utility*** u_{ij} of node i to node j
- The ***utility*** of a *seed set* S to a node j:

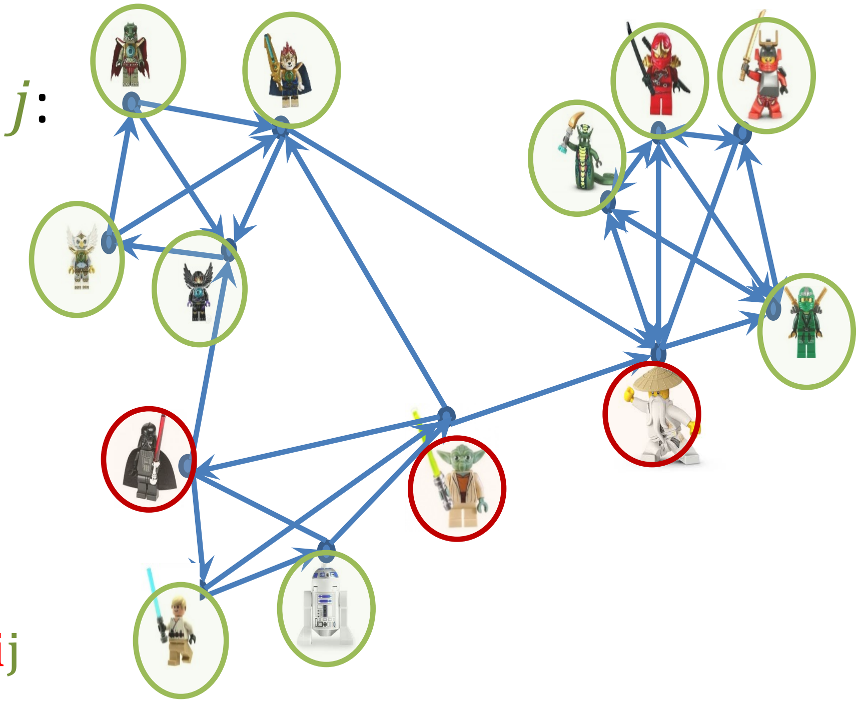
$$u_{Sj} = \operatorname*{aggregate}_{i \in S} \mathrm{u_{ij}}$$

e.g. $\text{aggregate} = \max$

- The ***influence*** of seed set S is the sum over j of the utility of S to j

$$\mathrm{Inf}(S) = \sum_j u_{Sj} = \sum_j \operatorname*{aggregate}_{i \in S} \mathrm{u_{ij}}$$

Centrality: Influence of a single node $\mathrm{Inf}(i) = \sum_j u_{ij}$



Aggregation functions

Utility of S to j

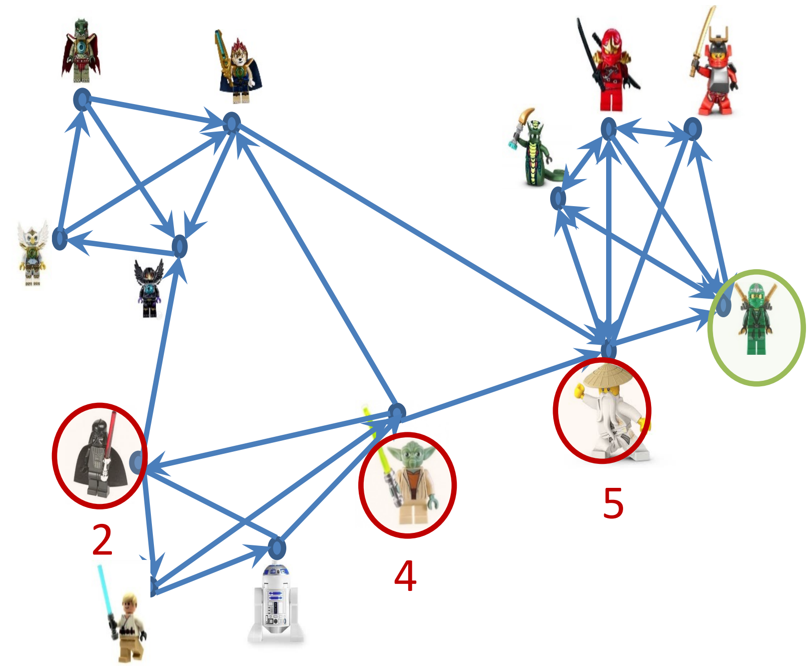
$$u_{Sj} = \underset{i \in S}{\text{aggregate}}\ \mathrm{u}_{\mathrm{ij}}$$

- Max: value equal to that the utility of the most useful seed node $u_{Sj} = 5$
- Sum: The more the merrier

  $$u_{Sj} = 5 + 4 + 2$$
- Top-2: sum of top two seed utility values $u_{Sj} = 5 + 4$ (limited capacity)

  +diminishing return $u_{Sj} = 5 + \frac{1}{2} \cdot 4$

$S = \{$  $\}$



Submodular top-ℓ : Up to top ℓ seeds contribute, non-increasing marginal contribution

$\Longrightarrow$ Influence function is submodular and monotone

When $|S| = 1$ (centrality): All "aggregate" are the same u_{Sj}

Pairwise utility from graph structure

Shorter paths, more paths, stronger edges on paths ⇒ higher u_{ij}

Ways to define utility u_{ij} from graph structure:

- *Reachability* $u_{ij} = 1 \Leftrightarrow i \rightsquigarrow j$ [Kempe Kleinberg Tardos 2003]++
- *Distance* $u_{ij} = \alpha(d_{ij})$ with decaying α [Bavelas 1948]++ [CK 2004] [Bloch Jackson 2007]++
  - Threshold: $u_{ij} = 1 \Leftrightarrow d_{ij} \leq T$ [Gomez Rodriguez et al ICML 11] [Du et al NIPS 13]++...
- ..... Reverse-rank $u_{ij} = \alpha(\pi_{ji})$ [Korn Muthu 01, Buchnik C 16]
- Survival time [C '16] (inspired by survival analysis)

\+ randomized models to generate edge lengths/presence
[Kempe Kleinberg Tardos KDD 2003, Gomez Rogriguez et al ICML 11, Abraho et al KDD13', Cohen et al COSN '13, Du et al NIPS '13]

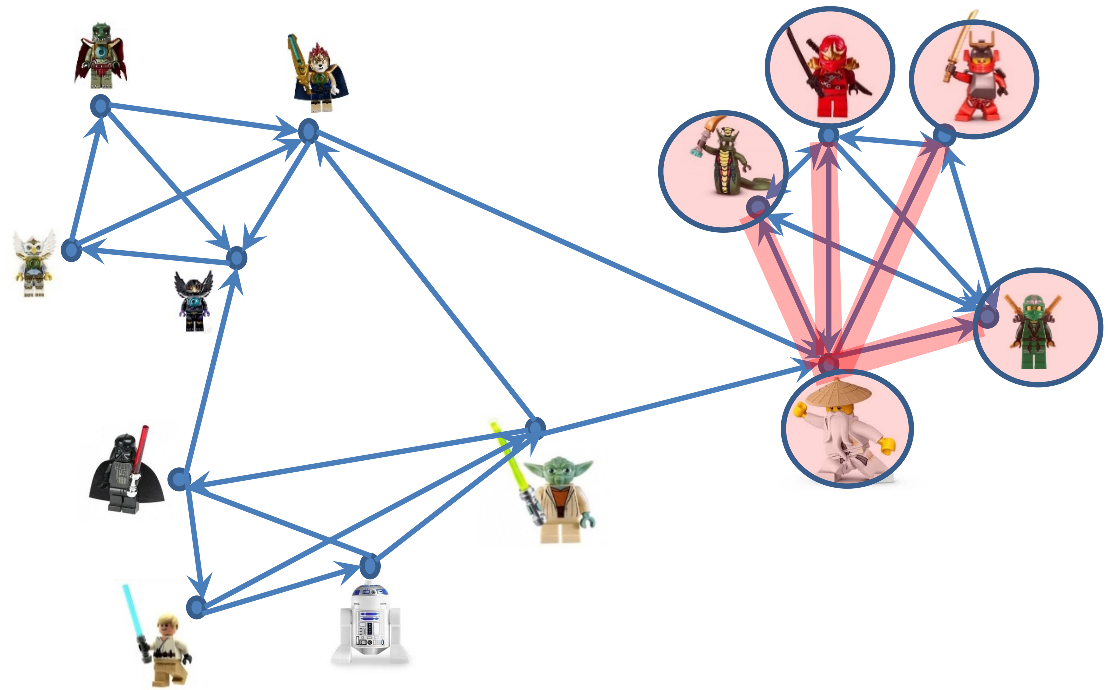
Simplest Model: Reachability

Utility: $u_{ij} = I_{j \rightsquigarrow i}$ aggregate=max: $u_{Sj} = \max_{j \in S} u_{ij} = I_{\exists j \in S \; s.t. \, j \rightsquigarrow i}$

$$\mathrm{Inf}(S) = \sum_{j} u_{Sj} \quad = \quad |\{i \mid \exists j \in S, j \rightsquigarrow i\}|$$

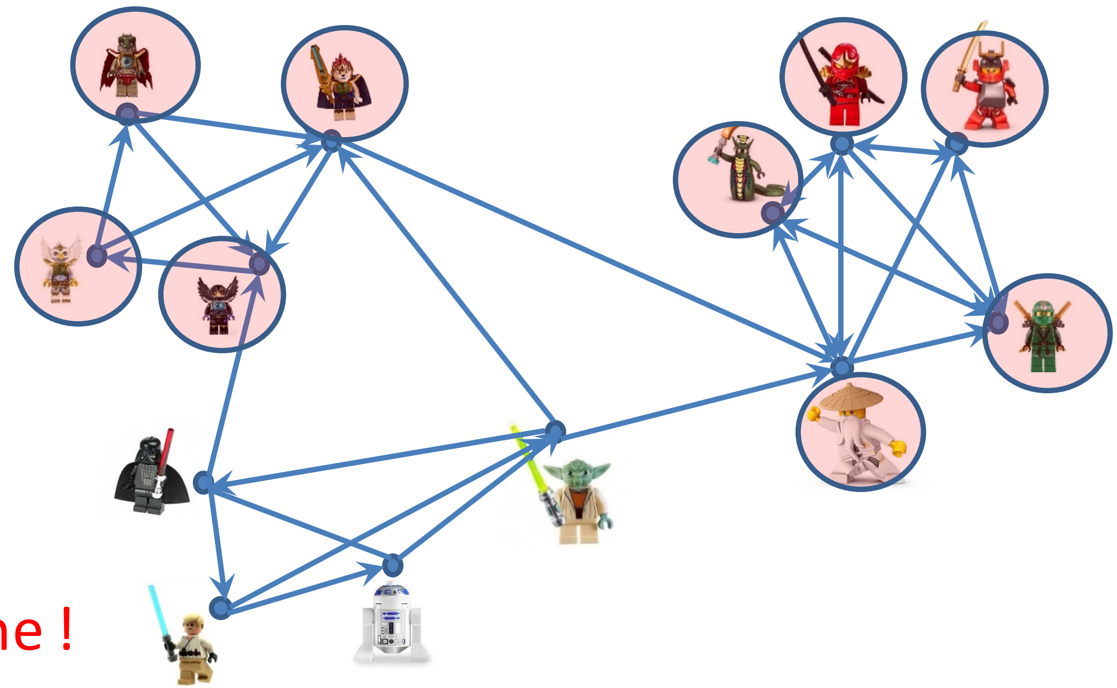
= #nodes reachable from at least one node in S.

$Inf($  $) = 5$



Simplest Model:
Reachability + max aggregation

$\mathrm{Inf}_{max}($   $) = 9$



Submodular and monotone !

Reachability + top-ℓ submodular aggregation

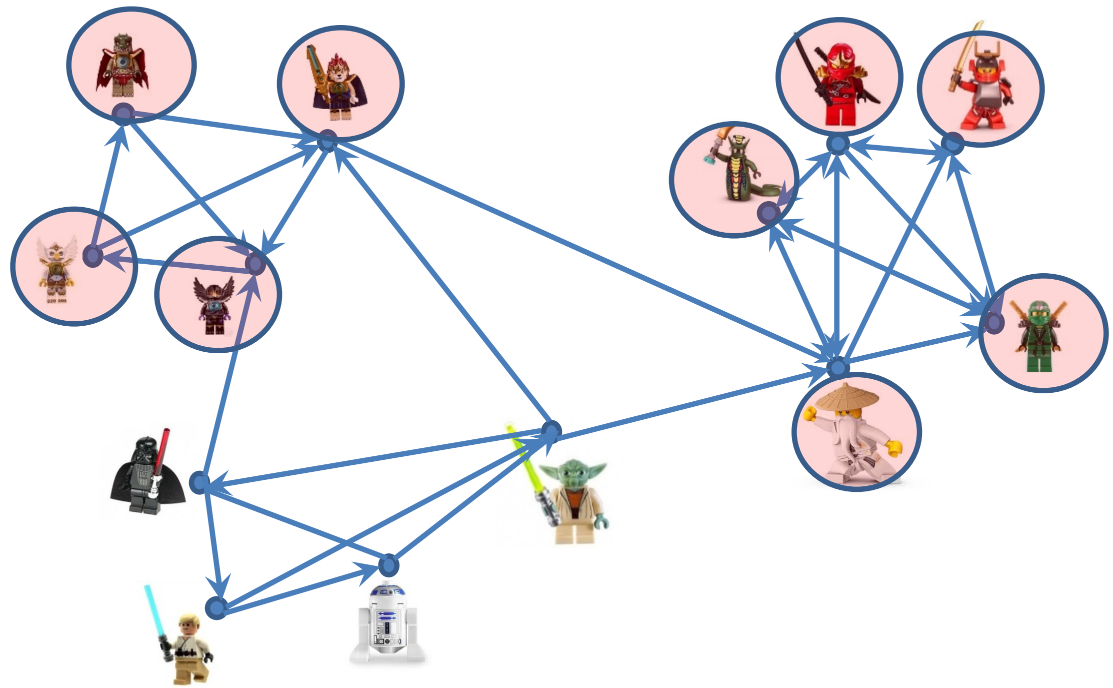
Utility: $u_{ij} = I_{j \rightsquigarrow i}$

$$\mathrm{Inf}(S) = \sum_j u_{Sj}$$

$$u_{Sj} = f(\#(j \in S\ s.t.\ j \rightsquigarrow i))$$

f monotone concave

$Inf($  $)$

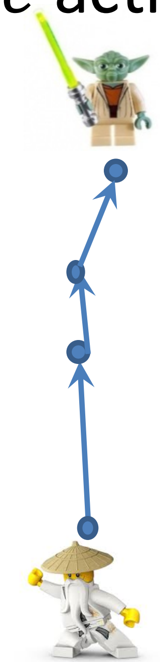


Randomized edge presence

Utility u_{ij} should decrease with path length and increase with path multiplicity.

Independent Cascade (IC) model [KKT '03]:

Edge e active with probability $\mathrm{p_e}$ (independent)

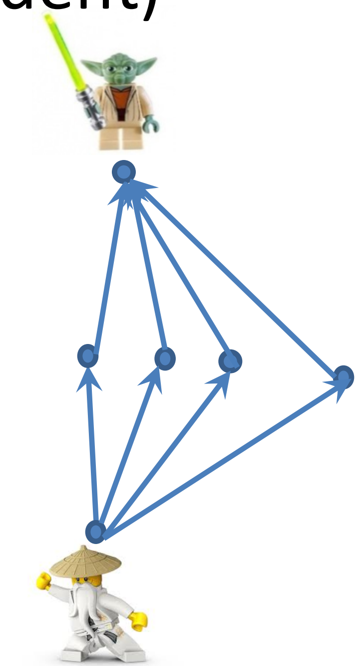


$deterministic$

$=$

$randomized$

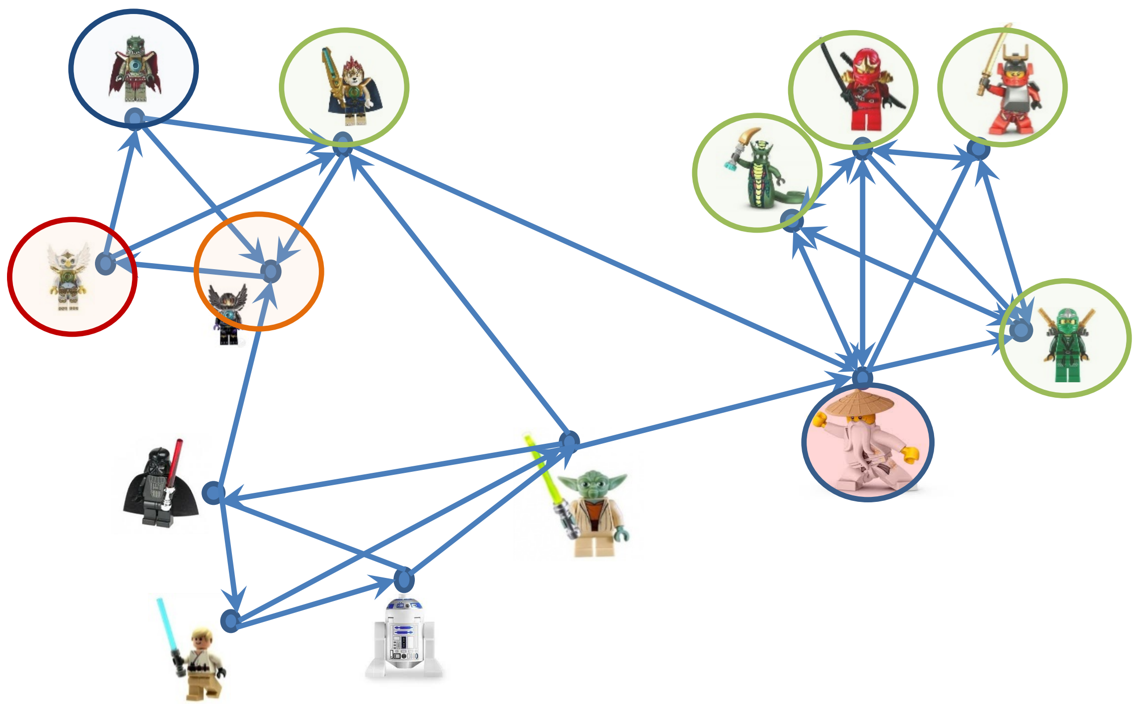
$\ll$



Distance-based Influence

Max aggregate

Utility: $\mathrm{u_{ij}} = \mathrm{e}^{-\mathrm{d_{ij}}}$ $\quad u_{Sj} = e^{-d_{Sj}}$ $\quad \mathrm{Inf}(S) = \sum_j u_{Sj}$



$Inf($  $) =$

$$1 + \frac{5}{e} + \frac{1}{e^2} + \frac{1}{e^3} + \frac{1}{e^4}$$

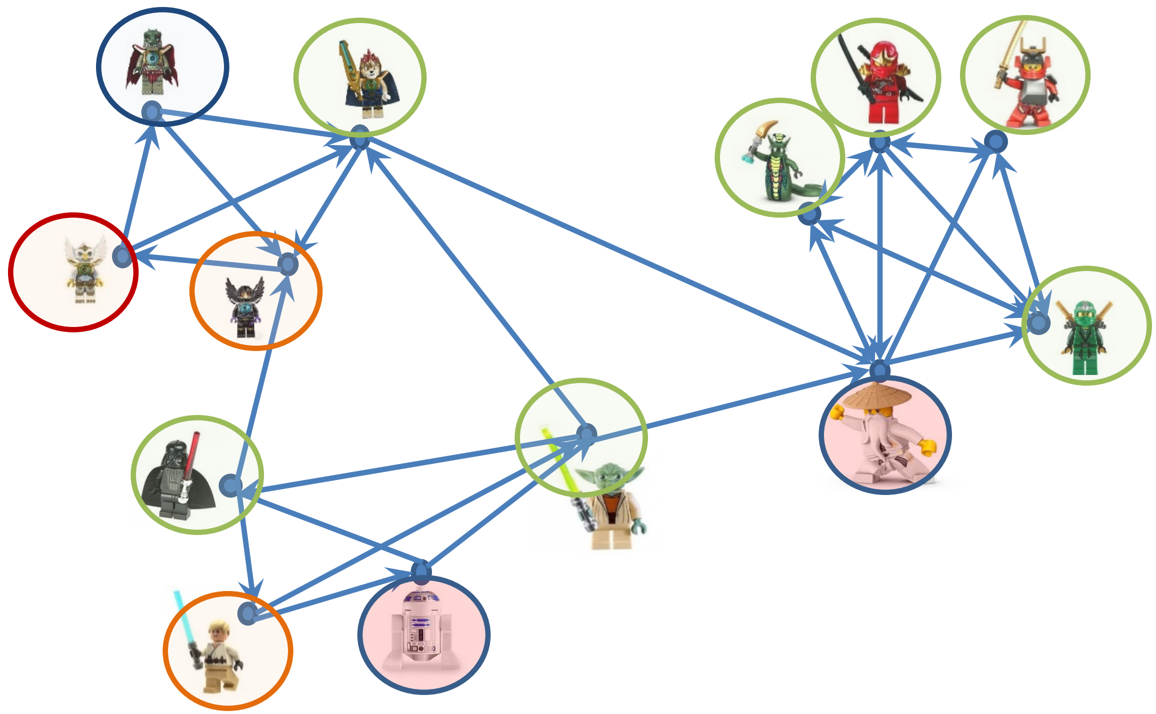
Distance-based Influence

Max aggregate

Utility: $\mathrm{u_{ij}} = \mathrm{e}^{-\mathrm{d_{ij}}}$ $u_{Sj} = e^{-d_{Sj}}$ $\mathrm{Inf}(S) = \sum_j u_{Sj}$

$Inf($  $) =$

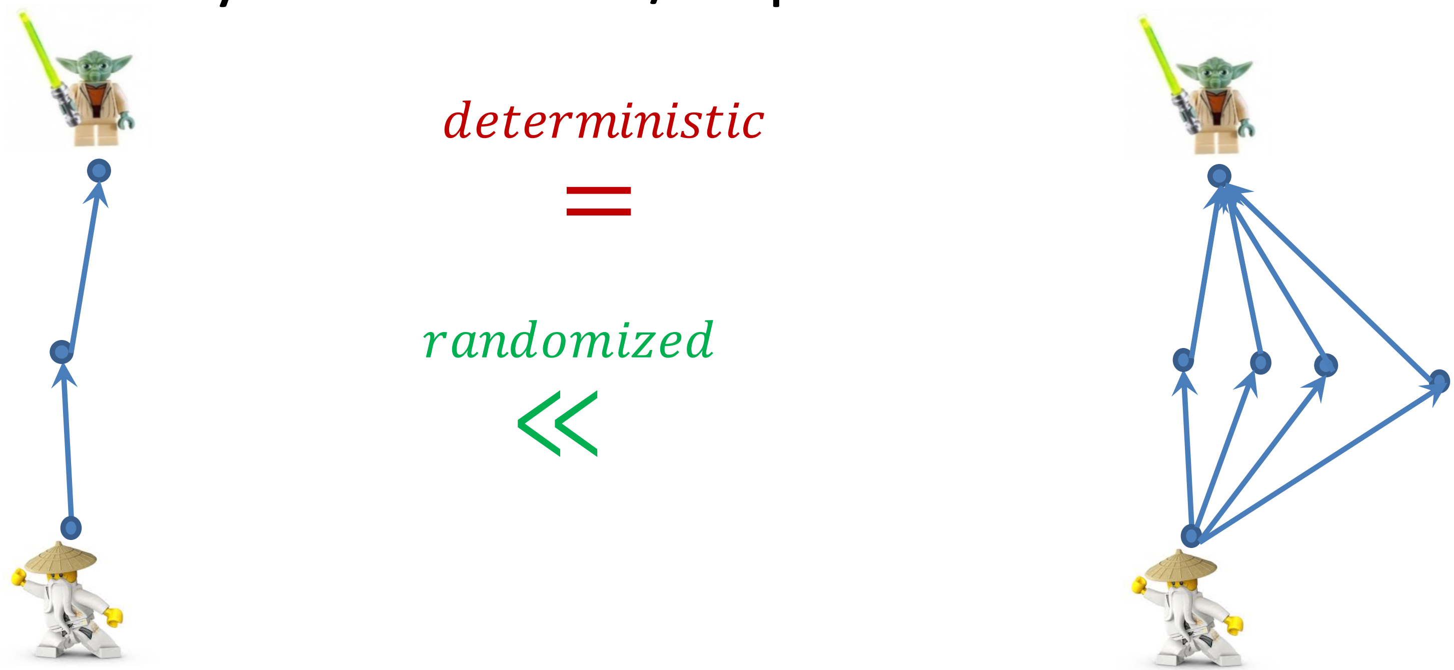
$$2 + \frac{7}{e} + \frac{2}{e^2} + \frac{1}{e^3} + \frac{1}{e^4}$$



Randomized edge lengths

Utility u_{ij} should decrease with path length and increase with path multiplicity.

Randomized: Edge lengths $\sim Exp[w_e]$ drawn independently from Weibull/ Exponential distribution



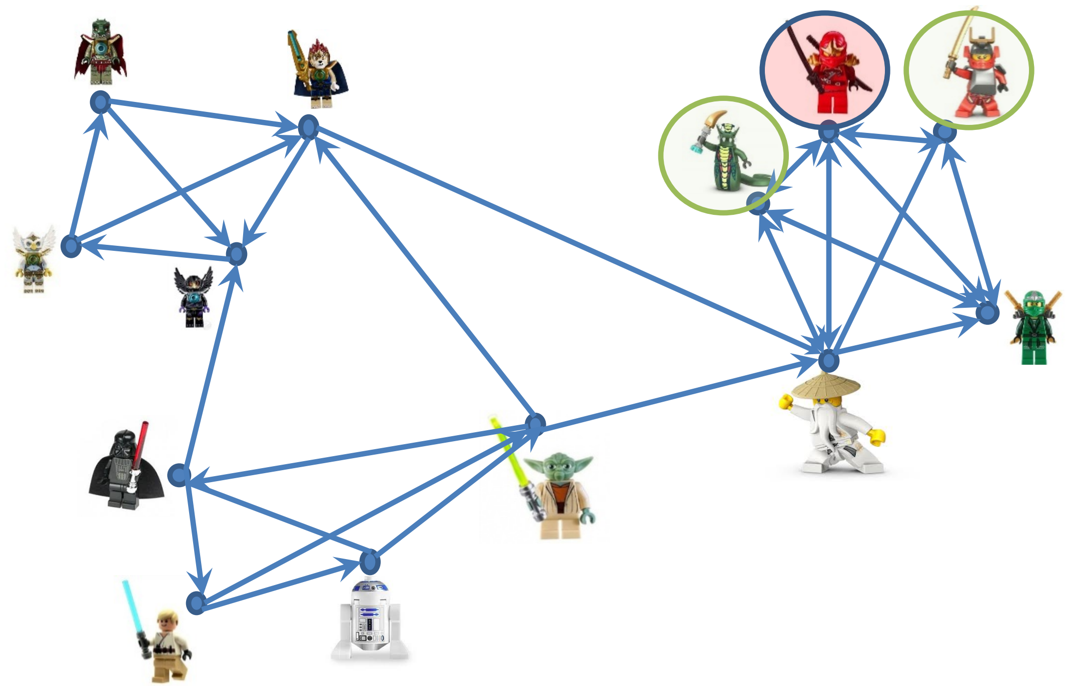
Reverse-rank Influence

(special case) reverse NN

Max aggregate

Utility: $u_{ij} = \mathrm{I}_{i=NN(j)}$ $\quad u_{Sj} = \mathrm{I}_{\mathrm{NN(j)}\in S}$ $\quad \mathrm{Inf}(S) = \sum_j u_{Sj}$

$Inf($  $) = 2$



Reverse-rank Influence

π_{ji} : position of i in increasing distances from j

Utility: $u_{ij} = \alpha(\pi_{ji})$

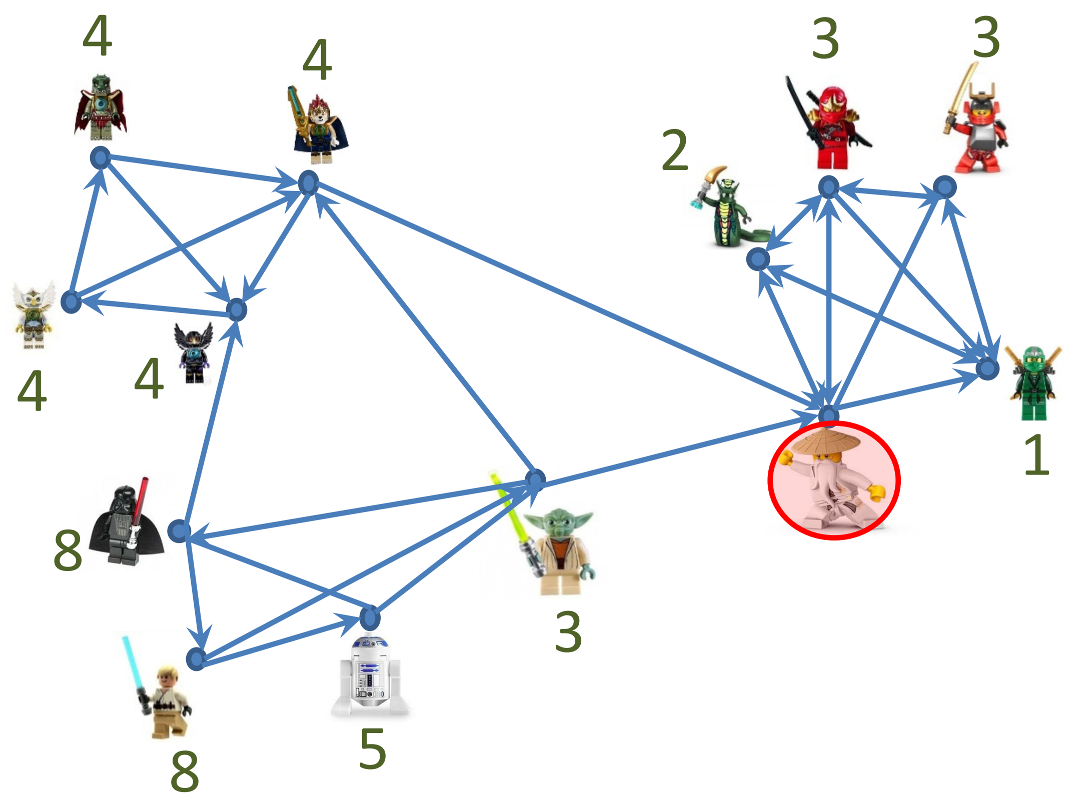
π_{ji} : rank of  by j

with $u_{ij} = \frac{1}{\pi_{ji}}$:

Inf() =

$$\mathbf{1} + \frac{\mathbf{1}}{\mathbf{2}} + 3 \cdot \frac{\mathbf{1}}{\mathbf{3}} +$$

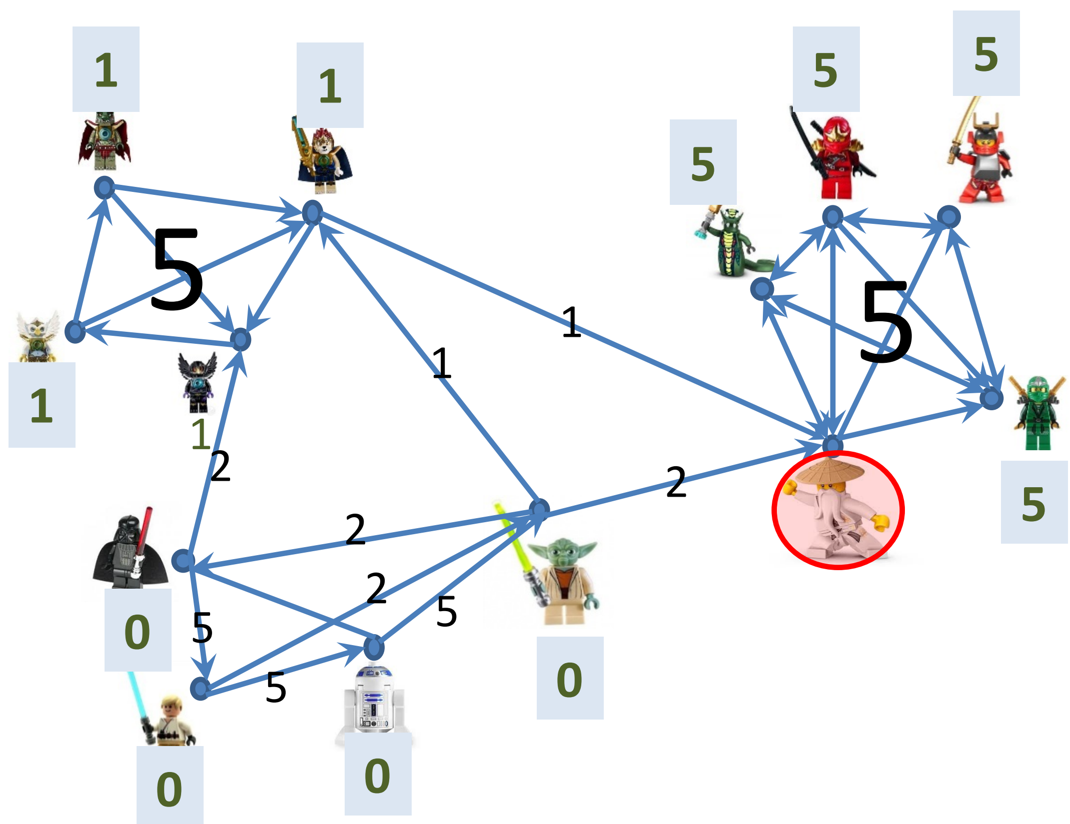
$$+4 \cdot \frac{\mathbf{1}}{\mathbf{4}} + \frac{\mathbf{1}}{\mathbf{5}} + 2 \cdot \frac{\mathbf{1}}{\mathbf{8}}$$



Survival time Influence

- Each edge has weight that corresponds to its "*lifetime*".
- t_{ij} maximum t such that $j \rightsquigarrow i$ when using edges with lifetime $\geq t$ (connectivity survival time)

Utility: $u_{ij} = t_{ij}$



Inf() =

$4 \cdot 5 + 4 \cdot 1$

Overview of contributions

- A unified model of graph-based influence functions: Includes functions proposed in previous work and extends to allow general submodular aggregations.
- A meta-algorithm for influence maximization: Modular design, near linear computation, statistical worst-case guarantees on approximation quality

Influence maximization

Given s, find a set of seed nodes S of size s with maximum influence $\arg\max_{|S|=s} \mathrm{Inf}(S)$

- NP hard, even to approximate [Feige '98]
- Influence function is submodular and monotone $\Longrightarrow$

Greedy algorithm is polynomial with approximation ratio

$$\geq 1 - \left(1 - \frac{1}{s}\right)^s > 1 - \frac{1}{e}$$ of opt [NWF '78]

Greedy iteration:

- Select $i \notin S$ with maximum $\mathrm{Inf}(S \cup \{i\}) - \mathrm{Inf}(S)$
- $S \leftarrow S \cup \{i\}$

- Greedy sequence approximates the full size/quality tradeoff
- But …Exact greedy too slow - need near-linear algorithms

Meta-SKIM
Sketch Based Influence Maximization

- Computes an *approximate greedy sequence*.
  Key property: Approximate the *marginal influence* of nodes to identify approximate maximizer *at each iteration*.
- Scales by maintaining and updating weighted samples (sketches) of marginal influence sets.

Meta-SKIM influence maximization

SKIM:

- reachability+max [CDPW ICDM'14]
- General decay + distances +max [CDPW '15]
- Threshold-decay+ reverse-rank + max [BC Sigmetrics '16]

Scales to networks with
10^8 edges, actual approximation within few percent

Meta-SKIM: unifies and generalizes previous work

- General decay (with distance, reverse rank, …)
- Reachability, distances, reverse ranks, survival threshold
- Submodular top-ℓ aggregations

Inherits scalability, statistical guarantees, computation bounds

Meta-SKIM

- Maintain weighted samples of “marginal influence sets” of nodes.
- Repeat:
  - Sample until estimates are accurate for “near-maximizers” of marginal influence.
  - Add the approximate maximizer to seed set.
  - Update residual problem

- Approximation ratio guarantee:

$$\geq 1 - \left(1 - \frac{1}{s}\right)^{s} - \epsilon \text{ times Opt}$$

- Near linear worst-case bound on computation !!

Meta-SKIM

- Maintain weighted samples of "marginal influence sets" of nodes.
- Repeat:
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Modular: Specified through oracle access to utility matrix

- Sampling uses reverse sorted access oracle: from j returns nodes in order of non-increasing utility u_{ij}. Implemented as graph search
- Updates use forward search oracle from a new seed. Returns all nodes with updated marginal utility. Also a graph search.

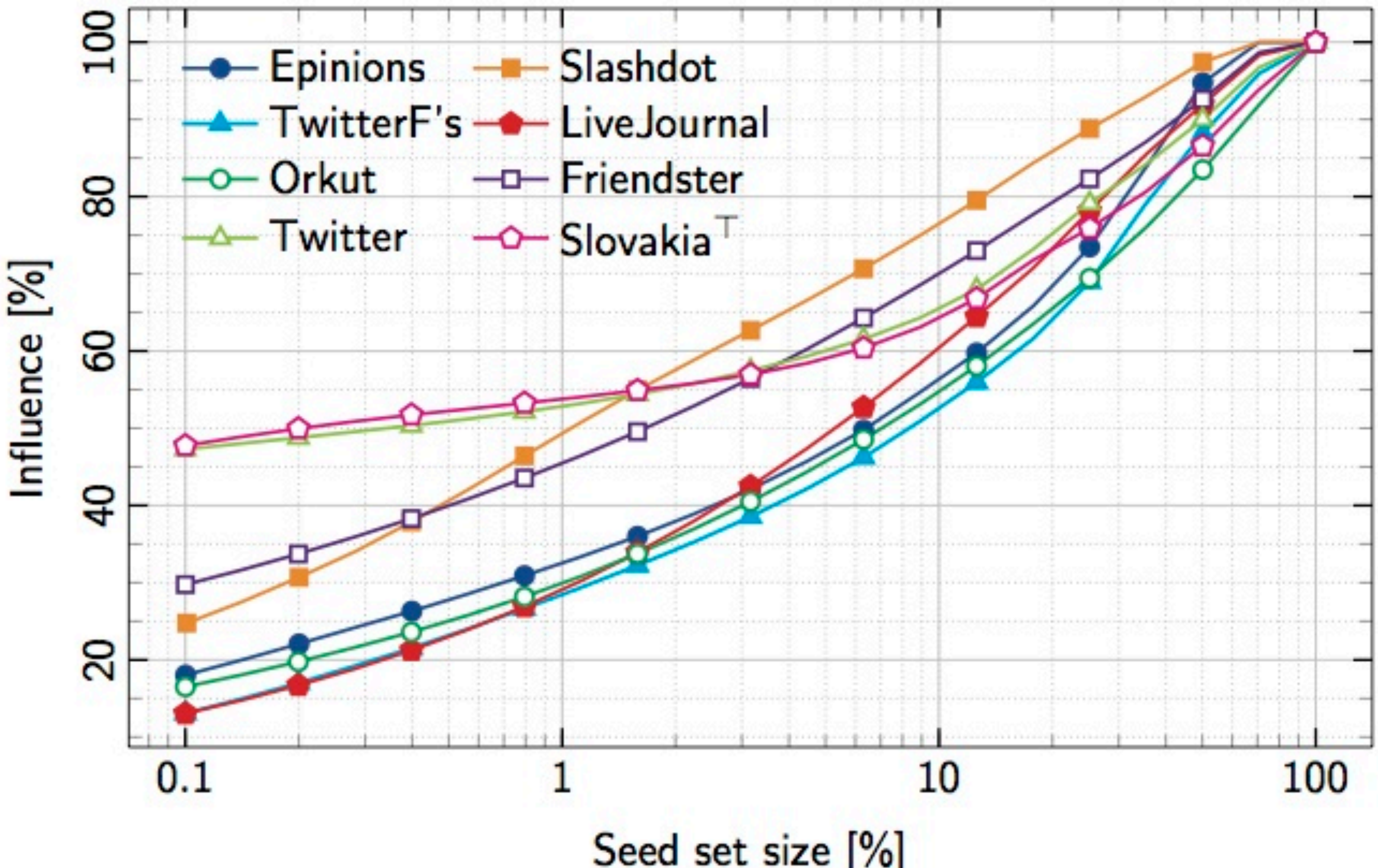
Meta-SKIM

- Maintain weighted samples of "marginal influence sets" of nodes.
- Repeat:
  - Sample until estimates are accurate for "near-maximizers" of marginal influence.
  - Add the approximate maximizer to seed set.
  - Update residual problem

Randomization handled using multiple MC simulations and optimizing for the average over simulations

Influence vs. #seeds: full approx greedy sequence

IC Model: Reach+ max aggregation + randomization

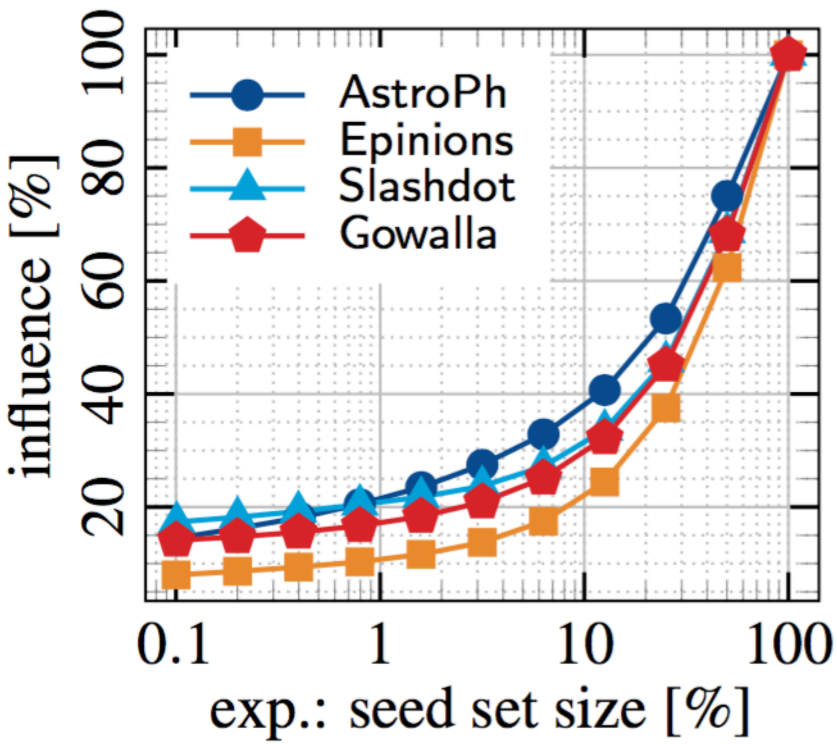


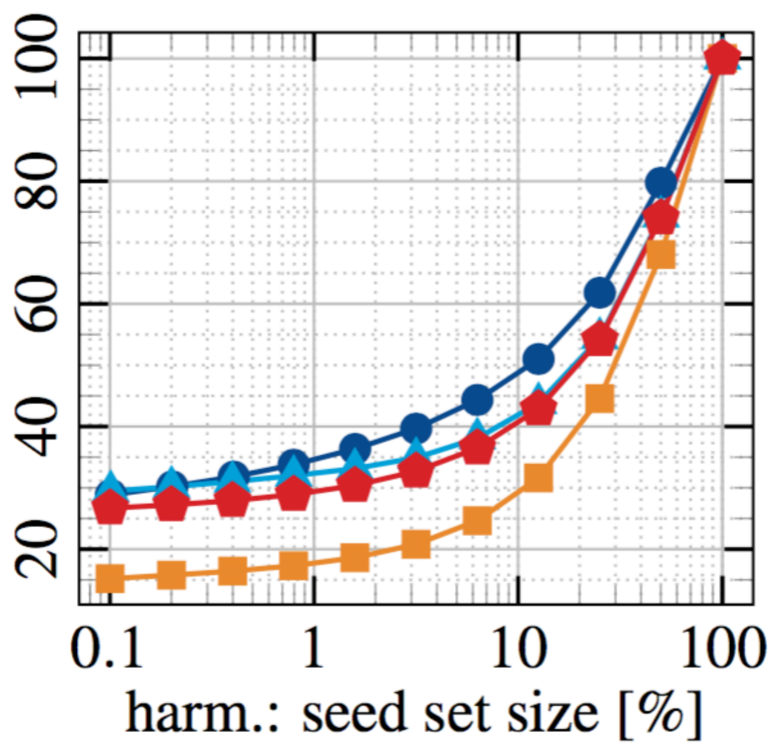
[CDPW ICDM 2014] data sets from SNAP

Implementation by T. Pajor (available)

Influence vs. #seeds: full approx greedy sequence

Distance utility with harmonic or exponential decay, max aggregation +randomization

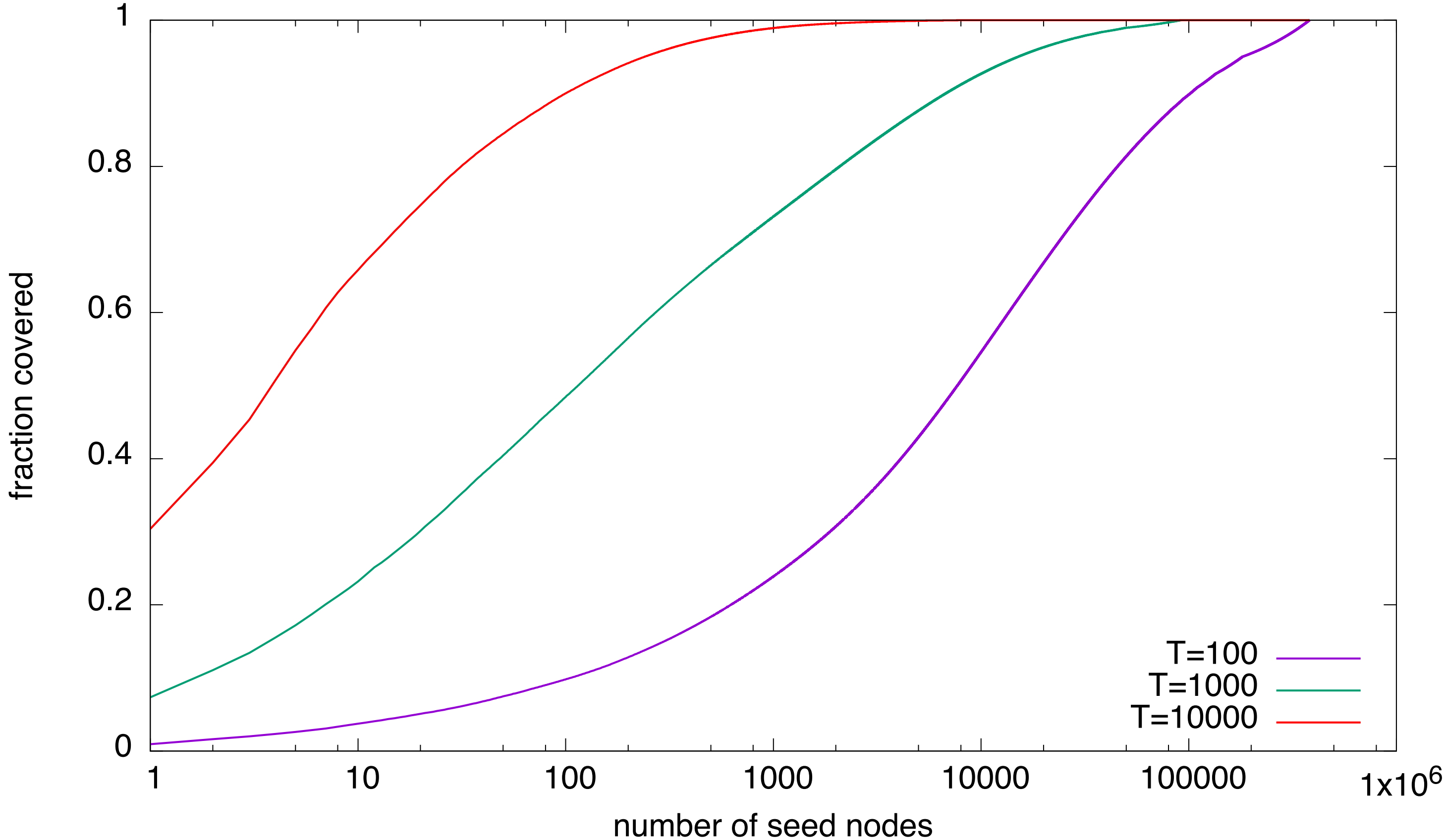




[CDPW 2015] data sets from SNAP

Implementation by T. Pajor

Influence vs. #seeds: full approx greedy sequence
Reverse Rank Utility, threshold decay (with different T)



[Buchnik C' Sigmetrics 2016] Live Journal data set from SNAP

Implementation by E. Buchnik (available)

Summary of contributions

- Unified model of graph-based influence functions
  - Influence functions specified by
    - pairwise *utility* values (reach/distance/reverse-rank/survival; decay function ; randomized generation)
    - Submodular aggregation function of seeds utility
- Meta-SKIM Algorithm: Compute an approximate greedy maximizing sequence using near linear computation for all functions

Follow up:

- Applications (seed selection for active learning,...)
- modular implementation

Thank you !!